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Strategic Design under Uncertain Evaluations: Structural Analysis of Design-Build Auctions

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Abstract

I investigate firms' competition over price and product design under uncertain design evaluations in the context of Design-Build (DB) auctions. Reviewers' design evaluations contain uncertainty from a bidder's perspective, leading *luck* to curtail differences in firms' chances of winning. I model bidders' behavior and derive semiparametric identification of the model primitives. Uncertain evaluations worsen the expected price of design quality, and exacerbate an auctioneer's uncertainty in auction outcomes. A simple adjustment in the auction mechanism may completely shut down the impact of uncertain evaluations on bidding incentives, restoring efficient allocations of projects.

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1 Introduction

Design competitions under uncertain evaluations are common in various settings. For example, academic researchers face uncertain evaluations of their grant proposals. Fashion designers face uncertain evaluations of their designs in a fashion contest. Public procurements that involve billions of dollars often solicit designs from consulting firms, and these competing firms also face uncertain evaluations of their designs by public officials.[1]

I study the effect of a client's uncertain evaluation on suppliers' design choices under strategic interactions. Transactions involving customized products require a design of the end-product by the supplier before the product is manufactured. However, suppliers typically do not know precisely what end-product their client would like.[2] Uncertain evaluations introduce an element of *luck* into design competitions, providing heterogeneous suppliers with heterogeneous incentives: good designers face a lower chance of winning from uncertain evaluations while bad designers face a higher chance of winning.[3] While it is clear that a project could be allocated to an inefficient contractor due to subjective design evaluations, it is not clear how competing suppliers respond to a change in the degree of uncertainty in a client's evaluation of their design proposals.

To study suppliers' behavior, I use hand-collected data on Design-Build (DB) auctions from the Florida Department of Transportation (FDOT). DB auctions are used not only by state departments of transportation in the U.S., but also in many other countries.[4] U.S. Department of Defense extensitvely uses DB to procure military construction projects and military weapons, which together cost the tax payers more than a hundred billion dollars annually.[5]

In a DB auction, bidders compete over price and design to win a contract to deliver an infrastructure project, ranging from bridge repair to building construction. Upon receiving price and design proposals, each reviewer of the FDOT independently evaluates and assigns a score to every design proposal. The quality score of a design proposal is then determined by

[1]Public Private Partnerships, which have surged in popularity among practitioners, are also an example of public procurement that involves a design competition among consulting firms.

[2]While suppliers may communicate with a client to reduce uncertainty, the client may be unwilling to do so since repeated interactions can be costly. Speed of delivery is often an important consideration in procurement.

[3]Note that a supplier faces uncertainty in the evaluation of its rivals' designs as well.

[4]As of October 2010, there are 39 state departments of transportation that use DB, including California, Delaware, Georgia, Minnesota, etc. DB auctions are also common in other developed countries, including Canada, Japan, and Sweden.

[5]The National Defense Authorization Act for Fiscal Year 2015 (NDAA) bill, which became public law in December 2014, explicitly prohibits the use of price-only auctions to procure construction services on military construction contracts. The Associated General Contractors of America, which consists of 26,000 construction firms, expresses its support to ban the use of price-only auctions for construction services for the reason that many quality aspects are ignored in price-only auctions.

the average across reviewers' evaluations. The bidder with the lowest price per quality score ratio (PQR) wins the project, and receives its price bid upon completing the project.[6]

The data reveal a substantial amount of discrepancy among reviewers for a given design proposal, and conversations with contractors confirm that uncertain design evaluations are a substantial concern among contractors. Such uncertainty in design evaluations, which I refer to as *evaluation uncertainty*, has not been considered to date in the vast auction literature. Indeed, to the best of my knowledge, there is virtually no empirical work that has investigated the implications of uncertain design evaluations on supplier behavior.

To guide the analysis, I develop a model in which each bidder strategically chooses its price and design quality in the face of uncertain design evaluations. The model allows for complex bidding strategies through multi-dimensional types: bidders are heterogeneous in the variable cost of providing a quality design, and in the fixed cost of implementing the designed project.

A large amount of uncertainty in design evaluation implies that project allocation is heavily influenced by luck. Inefficient bidders benefit from noisy evaluations since the bidders would otherwise lose in the absence of such a noise. Contrary to inefficient bidders, efficient bidders are less likely to win due to an increased contribution of luck. These asymmetric effects on bidding incentives result in (i) a higher expected price per unit of design quality, and (ii) a greater spread in price and design quality. That is, greater evaluation uncertainty worsens the expected price of design quality, and exacerbates the uncertainty in auction outcomes from the auctioneer's point of view.

Identification of the model is challenging. The econometrician does not observe bidders' design quality choices, but instead observes some noisy evaluations, precluding the standard inversion approach pioneered by Guerre, Perrigne, and Vuong (2000). Moreover, procurement auctions of infrastructure projects are known to contain a significant amount of unobserved auction heterogeneity: the cost commonly shared and observed by bidders but unobserved by the econometrician. Ignoring unobserved auction heterogeneity exaggerates the extent of bidder heterogeneities, which in turn undermines the effect of uncertain evaluations on bidding incentives. The degree of evaluation uncertainty could also be confounded with unobserved reviewer heterogeneity: differences in evaluations across reviewers that are inherent to reviewer specific characteristics. Ignoring unobserved reviewer heterogeneity exaggerates the extent of evaluation uncertainty since unobserved reviewer heterogeneity equally affects scores of different designs, leaving the design rankings unaffected.[7] Thus, I measure evaluation uncertainty by

[6]PQR is a winner selection rule used by many state departments of transportation, including Alaska, Michigan, North Carolina, and South Dakota.

[7]Unobserved reviewer heterogeneity may arise from reviewers having different quality standards. For ex-

the variation in idiosyncratic reviewers' evaluations net of unobserved reviewer heterogeneity. I define realization of such an idiosyncratic reviewer's evaluation as an evaluation noise.

I show semiparametric identification of the model in two steps. Using data on prices, reviewers' evaluations, and observables, I decompose bidding strategies into components explained by auction heterogeneity, bidder heterogeneities, reviewer heterogeneity, and evaluation noise. All the primitives, except for the distribution of bidder heterogeneities, are directly identified from this first step. Then, I combine the distribution of bids explained by bidder heterogeneities with bidders' first order optimality conditions to recover the distributions of bidder heterogeneities.

A large amount of evaluation uncertainty does not only affect bidding incentives, but also leads directly to an inefficient allocation of a project. While the auctioneer may wish to avoid such a misallocation of a project, reducing evaluation uncertainty through hiring a larger number of reviewers could be very costly. To circumvent this dilemma, I propose a simple auction mechanism in which each bidder submits a price per unit of design score, precluding evaluation noises from swapping the rankings of bids. The proposed auction mechanism restores the auction outcomes of a DB auction with no evaluation uncertainty. A counterfactual experiment, which examines a switch in the auction mechanism from the DB to the proposed mechanism, suggests substantial improvements in the auction outcomes.

The result of the analysis has an important economic implication for customized product markets. Subjective judgments of proposed designs may adversely affect clients due to both a misallocation of a project and the endogenous response of suppliers. The problem is particularly relevant when the client wishes to obtain a design "commonly" perceived as high quality, and not a design she herself likes. The client may wish to avoid such adverse effects by adopting a selection rule with some objective measures, such as price.

I build on and contribute to the growing literature on multi-attribute auctions and structural estimation of auction models. DB auctions are a particular type of a multi-attribute auction in which the winner selection rule is known to bidders.[8] Krasnokutskaya, Song, and Tang (2012) empirically investigate an auction environment in which the attributes-based winner selection rule is unknown to bidders.[9] In this paper, I argue that suppliers face uncertainty in evaluations

ample, reviewers' leniency in assigning a score may be captured by unobserved reviewer heterogeneity since a lenient reviewer tends to give a high score to every design proposal.

[8]The literature on multi-attribute auctions includes Athey and Levin (2001), Athey and Nekipelov (2012), Bajari *et al.* (2006), Bajari and Lewis (2011), Che (1993), Hanazono *et al.* (2012), and Nakabayashi and Hirose (2013), among others. There has been an extensive literature on structural estimation of first-price auction models, including Guerre, Perrigne, and Vuong (2000), Hendricks and Porter (1988), Laffont and Vuong (1993), Paarsch (1992), and Porter (1995).

[9]Krasnokutskaya, Song, and Tang (2012) make a distinction between multi-attribute auctions and scoring auctions based on whether or not the auctioneer's taste is observed. In this paper, I do not make this distinction,

of their innovative designs even if the selection rule is announced ex-ante.

The rest of the paper is organized as follows. Section 2 describes institutional details and the data. Sections 3 and 4 develop a structural model and derive identification of the model, respectively. Section 5 presents the estimation procedure and the results. Section 6 examines the effect of a change in mechanisms on the auction outcomes. Section 7 concludes.

2 Institutional details and data

This section describes the timing of events in a DB auction, institutional details critical for modeling and model identification, and stylized facts that indicate the economic significance of uncertain design evaluations.

2.1 Design-Build procurement auction

The procurement procedure can be decomposed into two consecutive stages: a pre-selection stage and a bidding stage. In the pre-selection stage, the FDOT posts an advertisement on-line which lists information about the project location, description of work, criteria for evaluating a letter of interest, and technical qualification requirements. Then, reviewers are selected from a pool of the FDOT employees by a department secretary, based on qualifications and availability. Meanwhile, an interested builder voluntarily matches with a designer, and writes a letter of interest to the FDOT. The appointed reviewers evaluate the letter of interest based on the criteria described in the advertisement, which include past performance grades of builders and designers, DB experience, and current capacity of builders. Then, pre-qualified applicants are short-listed and become "bidders".[10] The identities of these bidders are posted on-line and become common knowledge. The bidders then receive the request for proposal, which describes detailed specification of the project and design evaluation criteria.[11]

In the following bidding stage, all the bidders and the reviewers meet in a mandatory pre-proposal meeting in which the reviewers provide instructions and the scope of the project. Following the pre-proposal meeting, the bidders send a design and a price bid to the FDOT

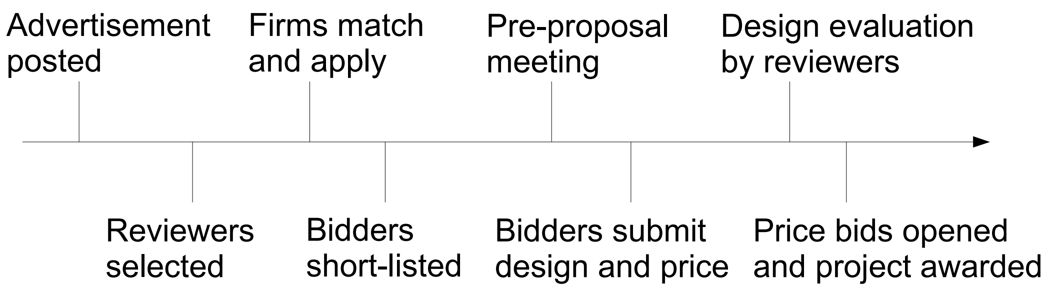
and treat them synonymously.

[10]There is no specific rule as to how many applicants should be short-listed, and the number of short-listed bidders ranges from two to five in the sample. The original set of auction records contains one auction with one bidder, and this auction is not used in any part of the analyses.

[11]Design evaluation criteria vary across auctions. Some repeatedly observed evaluation criteria include warranty, innovative aspect of design, maintenance of traffic, construction methods, commitment to environmental protection, project schedule, etc.

in separate envelopes.[12] The reviewers independently evaluate each design proposal, and the quality score of a design is determined by the average across reviewers' evaluations. Finally, the price bids are opened to determine the winner of the project based on price per quality score ratio (PQR).

Figure 1: Timeline of Events in a DB Auction



The institutional facts reveal important features from structural modeling perspectives. First, the reviewers are all employees of the FDOT. In addition, the compensation for appointed reviewers is salary based, and not based on each review task. Therefore, it is likely that the incentive to exert effort in reviewing tasks is weak, and could potentially contribute to noisy reviewers' evaluations.

Second, past record is an important factor for an applicant to be pre-qualified as a bidder. From a subset of DB records for which the identities of applicants are available, I find that the number of bidders significantly differs from the number of applicants. This observation is intriguing since decreasing competition through removing potential bidders would adversely affect the auctioneer. This particular observation may be explained by the opportunity cost of allocating reviewers to evaluation tasks. Indeed, these reviewers are highly skilled civil engineers who themselves are involved in the design of projects for standard procurement auctions of the FDOT.

Lastly and most importantly, the bidders do observe the reviewers who evaluate their designs in a pre-proposal meeting. While the presence of a pre-proposal meeting could imply that some of the uncertainty is resolved ex-ante, it is not clear how much information a bidder possesses about the reviewers at the time of bidding. Knowing the identities of reviewers is meaningful to a bidder only if some pattern or tendency can be inferred from the reviewers' identities. The

[12] Both design and price bids are usually due one to two months after the pre-proposal meeting.

sample of DB projects shows that the majority of reviewers are appointed only once in a decade and thus, the bidders are unlikely to make an inference about reviewers' characteristics from their past evaluations.

Table 1: Number of Times Bidders and Reviewers Appear in DB Auctions

Participant	Mean	Std	Min	Max
Builder	3.61	3.68	1	19
Designer	3.05	3.23	1	18
Reviewer	1.68	1.68	1	16

The sample contains 110 DB auctions procured between years 2000 and 2011. In total, there are 53 builders, 64 designers, and 250 reviewers in the sample.

The pre-proposal meeting also casts doubt on the incentive of bidders to lobby reviewers. Table 1 shows how frequently a particular reviewer is observed in the sample. On average, a reviewer appears in less than two auctions, and most reviewers appear only once in a decade. Thus, bidders play a one-shot game rather than a repeated game if they are to connect to a particular reviewer. While the one-shot nature of the game may not preclude bidders' incentive to lobby reviewers, its scope can be significantly limited by reducing the benefit of establishing a reviewer specific connection. Further, it is difficult for a reviewer to ensure the success of a particular bidder. While the reviewer could raise the chance of winning for a particular bidder by enlarging the gap in quality scores between the lobbying bidder and others, the reviewer cannot ensure that the bidder wins, since he/she does not know how other reviewers evaluate the design and what the price bids are.

In short, the reviewers from the FDOT have a weak incentive to exert effort, and have little experience in evaluating design proposals. Past performance is an important determinant of pre-qualification status, and review tasks are costly for the FDOT. As a bidder meets with other bidders in a pre-proposal meeting, every bidder knows who participates. While bidders also meet with reviewers before deciding on their bids, knowing the identities of reviewers is unlikely to reduce uncertainty in reviewers' evaluations.

2.2 Data

The data consist of a sample of DB auctions that took place in Florida between 2000 and 2011. Although DB is also a common practice in other states, scoring rules and point systems differ across these state departments of transportation. Therefore, a single department of transportation, the FDOT, is chosen for consistency and auction record availability.

The data used in the analysis are a subset of all 152 auction records provided by the FDOT. In particular, auctions with only one bidder, those missing FDOT's engineer's estimate of project cost, a modified scoring rule, or those missing reviewers' evaluations are all excluded from the data.[13] The selected sample is complemented by auction and bidder characteristics, which are obtained from bid tabs and through web-scraping.[14] Consequently, I am left with 110 auctions with detailed information on design evaluations. Out of the 42 excluded auctions, 28 auctions are removed for having a different auction rule, 11 auctions are removed for missing the FDOT engineers' estimates, two auctions are removed for missing individual reviewer level evaluation scores, and one auction is removed for having only one participating bidder.

Figure 2 is a particular record of design evaluations for a bridge construction project, and is one of the auctions with a large spread in design evaluations across reviewers in the sample. The first and second rows of the table shows the identity of three bidders and five reviewers, respectively. The first and second columns show 10 evaluation categories and weights. Each reviewer independently reviews each quality aspect of a design proposal, and assigns a score out of the category specific maximum score. Then, these scores are summed across all categories to obtain the total score of a design proposal, which I define as a reviewer's evaluation. These total scores are averaged across reviewers to determine the quality score of a bidder's design proposal. The three bidders are ranked by their PQR, and the bidder with the lowest PQR wins the project. A large variation in reviewers' evaluations can be easily verified from the reviewers' evaluations. For example, the difference in total scores assigned by JD and DK is 92-68=24 points for Cone & Graham/Jacob, which is 24% of the maximum allowable points. Also, JD ranks Cone & Graham/Jacob fifth and Johnson Bros./GAI third, while DK ranks Cone & Graham/Jacob first and Johnson Bros./GAI fourth.

Table 3 shows the summary statistics of the key variables. Prices are adjusted for inflation, and are expressed in 2011 USD. The average winning price is more than 16 million USD.

[13] The original dataset contains a variant of DB auctions in which the scoring rule involves a time incentive component. The variant of DB auctions is a combination of DB and A+B auction studied in Bajari and Lewis (2011).

[14] The set of auction and bidder characteristics include an FDOT's engineer's estimate of project cost, project types, work location, builder's closest branch to the work location, etc. It is well known that an engineer's estimate of project cost is an important control for project size heterogeneity in procurement auctions of infrastructure projects.

Table 2: Evaluation Scores from E7E10 Barge Canal Bridge Design Build Project

Team	Max Allowed	Cone & Graham / Jacobs					Category Total / Total Possible	Johnson Bros. / GAI					Category Total / Total Possible	PCL / HDR					Category Total / Total Possible
Evaluator		MS	DH	JD	DK	LC		MS	DH	JD	DK	LC		MS	DH	JD	DK	LC	
Environmental Protection/ Committments	9	7	8	6	8	7	36 of 45	7	6	8	7	8	36 of 45	7	7	8	6	7	35 of 45
Maintainability	14	12	10	11	13	13	59 of 70	12	8	10	11	12	53 of 70	11	9	5	11	10	46 of 70
Schedule	9	8	6	8	8	8	38 of 45	7	8	6	5	7	33 of 45	8	7	9	9	8	41 of 45
Coordination	9	7	7	8	8	9	39 of 45	7	6	8	8	8	37 of 45	7	5	8	8	8	36 of 45
Quality Management Plan	9	7	7	5	9	9	37 of 45	6	6	6	5	6	29 of 45	6	8	4	8	7	33 of 45
Maintenance of Traffic	4	3	4	2	3	4	16 of 20	3	4	2	2	3	14 of 20	3	4	2	4	3	16 of 20
Aesthetics	4	3	4	3	3	4	17 of 20	3	2	3	3	4	15 of 20	3	2	2	2	3	12 of 20
Design and Geotech Svcs Investigation	14	12	10	6	14	13	55 of 70	12	10	10	9	13	54 of 70	11	8	6	12	11	48 of 70
Construction Methods	14	12	11	7	13	12	55 of 70	12	9	12	11	12	56 of 70	12	10	9	14	11	56 of 70
Permit Acquisition	14	12	11	12	13	13	61 of 70	12	10	13	12	13	60 of 70	12	11	13	13	12	61 of 70
Total Score	100	83	78	68	92	92	**413 of 500**	81	69	78	73	86	**387 of 500**	80	71	66	87	80	**384 of 500**
Ordinal		1	1	2	1	1		2	3	1	3	2		3	2	3	2	3	

Evaluators:
MS = Reviewer 1
DH = Reviewer 2
JD = Reviewer 3
DK = Reviewer 4
LC = Reviewer 5

Table 3: Summary Statistics of Key Variables

	Mean	Std	Min	Max	Obs
Winning PQR ($1,000 / score point)	193	266	3.125	1125	110
Winning Price ($1,000,000)	16.6	22.8	0.253	103	110
Winning Quality Score (score point)	86.3	5.57	69.7	95.5	110
# Bidders / Auction	3.12	0.534	2	5	110
# Reviewers / Auction	3.82	0.800	3	6	110
PQR ($1,000 / score point)	249	346	3.12	1945	338
Price ($1,000,000)	20.9	28.6	0.253	142	338
Reviewer's Evaluation (score point)	84.3	8.19	38.6	100	1296

The summary statistics is calculated based on 110 DB auctions procured between years 2000 and 2011.

Considering the fact that the average winning price in usual first-price low-bid auction is 7.4 million USD in Florida, DB auctions seem to be adopted for relatively large scale projects. A quality score is the average across reviewers' evaluations, which is the weighted sum of category level scores. The weight assigned to an evaluation category varies across auctions even for the same evaluation category. Since the maximum quality scores vary across auctions, every quality score is standardized by its maximum possible score, and expressed out of 100 points. Note that rescaling of quality scores does not introduce any problem to the analysis since the winner selection rule is based on price per quality score, which is scale invariant to auction specific rescaling.

Table 4: Distribution of Winning Price and Quality Score

	Lowest Price	Non-Lowest Price	Total
Highest Quality Score	38	19	57
	(34.5%)	(17.2%)	(51.8%)
Non-Highest Quality Score	51	2	53
	(46.3%)	(1.8%)	(48.1%)
Total	89	21	110
	(80.9%)	(19.1%)	(100%)

Each row represents the frequency distribution of the project winners with the (non-)highest quality score. Each column represents the frequency distribution of the project winners with the (non-)lowest price.

Table 4 shows how many winners received the non-highest design quality score, and how many winners bid the non-lowest price. It is clear that neither lowest price bidder nor highest quality score bidder always win. Indeed, the majority of the winners do not receive the highest quality score.

To see how much variation exists in price and reviewers' evaluations, consider the following simple decomposition of variance in the natural logarithm of price, reviewer's evaluation, and price per reviewer's evaluation. Table 5 shows the decomposition of variance in the above three variables into between-auction, within-auction-between-bidder, and within-bidder-between-reviewer. The most significant finding here is that the within-bidder-between-reviewer variation is by far the largest contributor to the total variation in reviewers' evaluations. Within-auction-between-bidder variation in reviewers' evaluations, which may capture the degree of vertical design quality differentiation, is much smaller, and accounts for 31% of the total variation of reviewers' evaluations. The relatively large within-bidder-between-bidder variation in reviewers' evaluations may be indicative of the size of evaluation uncertainty that bidders face at the time of bidding.

Table 5: Variance Decomposition of Price Bids and Reviewers' Evaluations

Variables	Between-Auction	Within-Auction Between-Bidder	Within-Bidder Between-Reviewer
ln(Price)	1.43	0.158	
	(90%)	(10%)	
ln(Reviewer's Evaluation)	0.0389	0.0516	0.0779
	(23%)	(31%)	(46%)
ln(Price / Reviewer's Evaluation)	1.43	0.170	0.0779
	(85%)	(10%)	(5%)
Obs	110	338	1296

The figures in the table represent estimates of standard deviations. Standard deviations are calculated at each level of hierarchy, and a column corresponds to a particular level in the hierarchy.

3 Structural model of a DB auction

I construct a model where bidders compete over price and design quality under uncertain design evaluations and uncertain rivals' bids. The model incorporates three types of uncertainty: i) a bidder is uncertain about what its rivals' bids are, ii) how its design proposal is evaluated, and iii) how its rivals' designs are evaluated.

An efficient bidder, who submits a low PQR, experiences a lower probability of winning, while an inefficient bidder, who submits a high PQR, may win with a higher probability upon an increase in evaluation uncertainty. The asymmetric effects of evaluation uncertainty on bidding incentives across different types of bidders, together with strategic uncertainty, makes the equilibrium effects of evaluation uncertainty on bidders' behavior unclear.

In addition to strategic uncertainty, the model introduces multi-dimensional types in a bidder's cost structure in order to account for complexity in bidding strategies. Suppose that a bidder's cost consists of a fixed cost of implementing a project, and a variable cost of providing a quality project. On one hand, a bidder consisting of a low fixed cost and a high variable cost may take a low-price-low-quality strategy. On the other hand, a bidder with a high fixed cost and a low variable cost may take a high-price-high-quality strategy, exploiting their comparative advantages. However, these two completely different types of bidders may end up with very similar objective PQR bids, and generating complex bidding strategies is non-trivial without multi-dimensional types.

In the following, the multi-dimensional choice problem of a bidder with multi-dimensional types is reduced to a one dimensional choice problem of a bidder with a single index, as in Asker and Cantillon (2008). A Bayesian Nash Equilibrium of the model is characterized, and a numerical exercise is provided to shed a light on how bidders respond to a change in evaluation uncertainty.

3.1 Model

Consider $N \equiv |\mathcal{N}|$ risk neutral bidders where $\mathcal{N}$ denotes the set of bidders in a given auction. For the sake of notational simplicity, I suppress the auction subscript a in this section. Let $\{p_i, q_i\} \in \mathbb{R}_+^2$ be the price bid, and the objective quality of the design proposed by bidder $i \in \mathcal{N}$, respectively. Also, let $b_i \equiv p_i/q_i$ be the objective PQR bid of bidder i, which is assumed to be responsive only within the support $[0, B] \subset \mathbb{R}_+$, and the government rejects proposals outside the bounds.[15] To capture bidders' comparative advantage in designing, bidder i is characterized by a variable cost type, vc_i, and a fixed cost type, fc_i.

Now, define the ex-post payoff of bidder i by:

$$\pi_i^{post} = \begin{cases} p_i - vc_i\, C(q_i) - fc_i & \text{if bidder } i \text{ wins} \\ 0 & \text{otherwise,} \end{cases}$$

where $vc_i\, C(q_i)$ and fc_i consist of variable and fixed cost of delivering the project at quality level q_i. An assumption implicitly made here is that a bidder is committed to provide the quality it proposed with no quality shading.[16] $C(.)$ is increasing, convex, and differentiable (i.e., $C_q > 0$, $C_{qq} > 0$). The convexity is necessary to generate a smooth substitution between price and design quality given the winner selection rule. $C(.)$ is also common across bidders.[17] Let

[15] An interpretation of the boundedness assumption is that the government does not accept a bid that goes above the FDOT's reserve price, which represents FDOT's willingness to pay for the project at hand.

[16] FDOT's engineers monitor construction progress, and a contractor that does not deliver the planned project properly may not pre-qualify for subsequent design-build projects.

[17] Later in specifying the econometric form of the model, I allow for vc_i and fc_i to consist of observed and unobserved components from the point of view of bidder i's rivals. In this section, I assume that vc_i and fc_i are entirely private information of bidder i in this section without loss of generality, and for expositional simplicity.

$c_i \equiv \{vc_i,\ fc_i\} \in T_i \subset \mathbb{R}^2_+$ and $\mathbf{c_{-i}}$ denote the vector of bidders' types in the auction excluding bidder i. Bold cases are used for vectors (e.g., $\mathbf{b} \equiv (b_1, , , b_i, , , b_N)$, $\mathbf{c_{-i}} \equiv (c_1, , , c_{i-1}, c_{i+1}, , , c_N)$). Also, let $f(\mathbf{c_{-i}}|\ c_i)$ denote the joint distribution of $\mathbf{c_{-i}}$ conditional on the realization of bidder i's type, which is differentiable everywhere over its compact support.

The specification of the ex-post payoff function assumes zero entry costs while preparation of a design could be costly in reality. In other words, the model assumes the major cost of providing a quality project comes from implementation of a quality project, and not designing itself. Zero entry cost assumption here could alternatively interpreted as every bidder incurring the same amount of entry cost, but compensated by the auctioneer for the entry cost. Indeed, the FDOT provides bidders with stipend to compensate for the cost of preparing a design proposal in many DB auctions.

Define evaluation noise as the total amount of subjectivity that a set of reviewers introduce into a quality score of bidder i's design proposal. Also, define evaluation uncertainty as the degree of dispersion in evaluation noise. Assume that evaluation noise, w_i, generates realization of a quality score by multiplicatively affecting q_i, and is independently distributed from design quality. Let $F_w(\mathbf{w})$ be the joint distribution function of evaluation noise w_i. The probability of winning conditional on a vector of PQR bids, $\tilde{G}_i(\mathbf{b_i})$, is given by:

$$\begin{aligned}
\tilde{G}_i(\mathbf{b}) &= \int_{\mathbf{w}} \mathbf{1}\{\text{bidder } i \text{ is the winner given } \boldsymbol{b}\} dF_w(\mathbf{w}) \\
&= \int_{\mathbf{w}} \mathbf{1}\left\{\frac{p_i}{q_i w_i} < \frac{p_j}{q_j w_j}\ ,\ \forall\, j \neq i\right\} dF_w(\mathbf{w}) \\
&= \int_{\mathbf{w}} \mathbf{1}\left\{ln(b_i) - ln(w_i) < ln(b_j) - ln(w_j)\ ,\ \forall\, j \neq i\right\} dF_w(\mathbf{w}).
\end{aligned}$$

As bidder i does not observe the private information of its rivals, bidder i's probability of winning is obtained by integrating $\tilde{G}_i(\mathbf{b})$ over the distribution of bidder i's rivals' strategies. Let $\boldsymbol{\psi_{-i}(c_{-i})}$ denote a vector of bidder i's rivals' PQR strategies. Then:

$$G_i(b_i, \boldsymbol{\psi_{-i}}) \equiv \int_{\mathbf{c_{-i}}} \tilde{G}_i(b_i, \boldsymbol{\psi_{-i}(c_{-i})})\, f(\mathbf{c_{-i}}|c_i)\, d\mathbf{c_{-i}}.$$

Finally, bidder i's interim expected payoff conditional on participation is defined as $\pi_i^{int} \equiv G_i(b_i, \boldsymbol{\psi_{-i}})\, \pi_i^{post}$. The problem of bidder i is then defined as:

$$\max\left\{\max_{p_i, q_i}\ \pi_i^{int}\ \ s.t.\ p_i/q_i = b_i \in [0, B],\ 0\right\}. \tag{1}$$

Let $\{p_i^{BR}(c_i), q_i^{BR}(c_i)\}$ be a best response correspondence of bidder i of type c_i with an arbitrary belief about its rivals' strategies. A Bayesian Nash Equilibrium is a state in which every bidder's belief is consistent with the best responses of its rivals. A Bayesian Nash Equilibrium is called "pure" if every bidder's strategy is a deterministic function of own type.

Definition 1. *A Pure Strategy Bayesian Nash Equilibrium consists of a profile of best response functions* $\{\mathbf{p^{BR}}(\mathbf{c}), \mathbf{q^{BR}}(\mathbf{c})\}$ *in which every bidder* $i \in \mathcal{N}$ *believes its rivals bid according to* $\{\mathbf{p^{BR}_{-i}}(\mathbf{c_{-i}}), \mathbf{q^{BR}_{-i}}(\mathbf{c_{-i}})\}$*.*

This two-dimensional decision problem can be transformed into a one-dimensional choice problem. Consider the optimization problem in (1) with an additional constraint that $b_i = \alpha$ for some $\alpha \in [0, B]$. This constrained optimization problem has a unique solution, and the values of price and objective design quality that solve this problem are given by the following closed form expressions:

$$q_i(\alpha) = C_q^{-1}(\alpha/vc_i) \tag{2}$$

$$p_i(\alpha) = q_i(\alpha)\, C_q^{-1}(\alpha/vc_i), \tag{3}$$

where $C_q^{-1}(.)$ is the inverse of $C_q(.)$.

Proposition 1. *For any given* $b_i = \alpha \in [0, B]$*, there exists a unique pair of* $\{p_i, q_i\} \in \mathbb{R}^2_+$ *that maximizes i's interim expected payoff conditional on participation.*

Proof in Appendix. This proposition establishes that pricing and design decisions are uniquely determined for any given PQR. It follows that the problem of a bidder can be rewritten as one-dimensional choice problem:

$$\max_{b_i \in [0,B]} \quad G_i(b_i, \boldsymbol{\psi_{-i}})\,(p_i(b_i) - vc_i\, C(q_i(b_i)) - fc_i),$$

where $p_i(.)$ and $q_i(.)$ are the functions defined in (2) and (3), respectively. Now, I make a assumption on $C(.)$ to summarize multi-dimensional types of a bidder with a single index.

Assumption 1. *The design cost function* $C(.)$ *is homogeneous of degree* $\gamma > 1$*.*

Assumption 1 implies scale invariance and allows for sorting of bidders in single index.

Proposition 2. *Given Assumption 1, equilibrium PQR strategy of bidder i is a sole function of a single index, $e_i \equiv fc_i / C_q^{-1}(1/vc_i)$:*

$$\begin{aligned} \psi_i(e_i) &\equiv \arg\max_{b_i \in [0,B]} \quad G_i(b_i, \boldsymbol{\psi_{-i}})\,(p_i(b_i) - vc_i\, C(q_i(b_i)) - fc_i) \\ &= \arg\max_{b_i \in [0,B]} \quad C_q^{-1}(1/vc_i)\, G_i(b_i, \boldsymbol{\psi_{-i}})\,(u(b_i) - e_i) \\ &= \arg\max_{b_i \in [0,B]} \quad G_i(b_i, \boldsymbol{\psi_{-i}})\,(u(b_i) - e_i) \qquad \forall\ i \in \mathcal{N}, \end{aligned}$$

where $u(b_i) \equiv b_i\, C_q^{-1}(b_i) - vc_i\, C(C_q^{-1}(b_i))$.

Proof in Appendix. The single index is an increasing function of vc_i and fc_i. Suppose, for example, that the quality cost function is a power function, such that $C(q) = q^\gamma$ with $\gamma > 1$. γ captures the weight assigned on vc_i and fc_i within e_i, and vc_i and fc_i would equally contribute to e_i when $\gamma = 2$ (i.e., $e_i = vc_i\, fc_i$). In the extreme case where γ is substantially larger than 2, e_i is essentially determined by fc_i. For the rest of the analyses, e_i is referred to as the efficiency type of bidder i.

The following technical assumptions on the distribution of evaluation noise w_i is made in order to simplify equilibrium characterization.

Assumption 2. *(Smooth Density and Independence): Log evaluation noise, $ln(w_i)$, is drawn independently from a smooth density with an infinite support.*

Assumption 3. *(Profitable Participation): Every bidder has a chance to make some profit (i.e., $B > u^{-1}(\bar{e})$ where $\bar{e}$ is the most inefficient bidder).*

Assumptions 2 and 3 together guarantee differentiability of the probability of winning function and participation of every bidder.[18]

Proposition 3. *Equilibrium Existence, Monotonicity, and Continuity: There exists a pure strategy Bayesian Nash Equilibrium. In any equilibrium, $\psi_i(e_i)$ is non-decreasing and continuous in e_i $\forall\, i \in \mathcal{N}$ in the interior of the domain.*

Proof in Appendix. Let $\{\mathbf{p}^\psi(\mathbf{c}), \mathbf{q}^\psi(\mathbf{c})\}$ be the corresponding price and design quality strategy profile in an equilibrium. If bidders' strategies are interior, which I assume for the rest of

[18]Differentiability of the probability of winning function with respect to own PQR bid is guaranteed regardless of rivals' bidding strategies since evaluation noise smooth out the probability of winning function even if rivals' bids are bunched. Also, participation is guaranteed since a bidder faces a positive probability of winning due to evaluation uncertainty no matter how large its PQR bid is.

the paper, the first order optimality condition from the above one-dimensional choice problem together with the ratio of (2) and (3) gives the following two equations:

$$\psi_i(e_i) = vc_i\, C_q(q_i^{\psi}(c_i)) \tag{4}$$

$$-\frac{u_\psi(\psi_i(e_i))}{u(\psi_i(e_i)) - e_i} = \frac{g_i(\boldsymbol{\psi}(\boldsymbol{e}))}{G_i(\boldsymbol{\psi}(\boldsymbol{e}))}, \tag{5}$$

where $g_i(\psi) \equiv \partial G_i(\psi)/\partial\psi$ and $\boldsymbol{\psi}(\boldsymbol{e})$ is an equilibrium PQR strategy profile.[19] Note that price per quality is the price a bidder charges for a unit of design quality. Thus, condition (4) says that bidder i's offer of price of design quality is set equal to the marginal cost of providing an additional design quality. This condition is intuitive. Suppose that $p_i/q_i < vc_i\, C_q(q_i)$ for some bidder i, such that bidder i is offering a unit of design quality at the price lower than its marginal cost. Then, bidder i could reduce both its price and design quality by proportionally the same amount to keep its price per quality constant, and yet bidder i can reduce its marginal cost, which must make bidder i better off. Condition (5) shows that the marginal cost of raising PQR is equalized to its marginal benefit, capturing the trade-off between the chance of winning and the ex-post winning payoff, which is analogous to the first order condition in a model of first-price sealed-bid auction in the independent private value paradigm.

Note that condition (4) is independent of the distribution of evaluation noise. If a bidder increases its PQR bid upon a change in the distribution of evaluation noise, then it responds by producing a design of higher quality, and there is no way to lower its price while raising both its PQR and its design quality. Proposition 4 formalizes this observation.

Proposition 4. *Let τ be any parameter that affects the winner selection outcome, but does not have any effect on bidders' exogenous costs. Then, sign $\left(\frac{d\,\psi_i(e_i)}{d\,\tau}\right)$ = sign $\left(\frac{d\,p_i^{\psi}(c_i)}{d\,\tau}\right)$ = sign $\left(\frac{d\,q_i^{\psi}(c_i)}{d\,\tau}\right)$. In other words, a cost-irrelevant parameter τ induces positive co-movements in bidders' strategies.*

Proof in Appendix. τ can be anything that influences strategic behavior, but does not enter a bidder's cost. For instance, suppose that τ represents the number of participating bidders in an auction. An intense competition may lower the price, but it may also deteriorate the quality, invoking a race to the bottom.

An important application of this proposition is when τ represents a measure of uncertainty in reviewers' evaluations. Suppose that a bidder increases its price per quality upon an increase

[19]Condition (4) is the ratio of the first order conditions with respect to price and design quality. I obtain condition (5) by (i) representing price and quality choices as a function of PQR strategy $\psi_i(e_i)$ in the expected profit function π_i^{int}, and then (ii) taking the first order condition with respect to ψ_i.

in evaluation uncertainty. An increase in its PQR bid implies that each unit of design quality is offered at a higher price. Therefore, the bidder strategically substitutes design quality for price, producing a design of higher quality at a higher price. The improvement in quality is not necessarily desirable from an efficiency perspective. Consider condition (4), which equalizes the average price of design quality with the bidder's marginal cost of providing design quality. As a bidder always sets the price per quality larger than its average cost of providing a design quality, the bidder's marginal cost is necessarily greater than the average cost. Therefore, every bidder overproduces design quality relative to its efficient scale under the DB auction rule. An increase in design quality comes with a larger average cost of design quality.

3.2 Numerical exercise

While a theoretical characterization of bidders' equilibrium behavior is difficult, a numerical exercise may shed a light on the effects of evaluation uncertainty on bidders' behavior of different types. I also demonstrate the equilibrium effect of evaluation uncertainty on the distribution of price and design quality bids. All of the parameter values in this numerical exercise are set equal to the estimates obtained from a structural estimation of the model.[20] As equilibrium uniqueness is not guaranteed in the model, an equilibrium is computed using a homotopy method. An algorithm for computing an equilibrium is provided in Appendix.

Figure 2 illustrates the effect of evaluation uncertainty on the probability of winning function keeping rivals' strategies constant. An increase in evaluation uncertainty *flattens out* the probability of winning function. There are two distinct channels where increased randomization affects the winner selection process.

The first channel is the level effect that is heterogeneous across different types of bidders. Suppose there is an exogenous increase in evaluation uncertainty. Inefficient bidders now expect to win the project with a higher chance since these bidders have little chance of winning without luck. Therefore, inefficient bidders have an incentive to shade their bids, and enjoy a higher payoff upon winning. In contrast to inefficient bidders, efficient bidders experience an exogenous decrease in their chance of winning, generating an incentive to lower its PQR bid to ensure that they win. Therefore, the level effect generates a greater dispersion of PQR bids and associated choice of price and design quality bids by Proposition 4.

[20] To keep the computation of equilibrium simple, I assume that $ln(w_i)$ follows Type 1 Extreme Value distribution. This assumption allows for a closed form expression for the probability of winning function $G_i(.)$ that resembles Tullock's Contest Success Function.

The second channel is the slope effect that is symmetric across different types of bidders. Since an increase in evaluation uncertainty lowers the marginal effect of lowering PQR on a bidder's chance of winning, it weakens a bidder's incentive to be competitive. Consequently, increased randomization in winner selection generates a higher PQR for all types of bidders. Figure 3 illustrates the equilibrium effect of evaluation uncertainty on the distribution of price and design quality bids.

The numerical exercise indicates that, on average, evaluation uncertainty increases price, but also improves design quality. In addition, an increase in evaluation uncertainty is associated with a larger dispersion in both price and design quality. The greater dispersion in bids in turn leads to a greater amount of uncertainty in the auction outcomes from the point of view of the auctioneer.

The above findings have some important economic implications. First, evaluation uncertainty, on average, can be seen as a transfer to the contractor. As luck plays a larger role in determining the winner of a project, bidders have less incentive to provide a competitive offer, leaving larger rents to bidders on average. Second, an increase in evaluation uncertainty comes with an additional cost of increased uncertainty in auction outcomes. If the auctioneer is budget constrained, an unexpectedly high winning price for a very large project may result in the cancellation of the procurement itself. Thus, a mechanism that reduces evaluation uncertainty may become a valuable option. I propose a simple auction mechanism that achieves this objective in Section 6.

Figure 2: The Effects of Uncertain Design Evaluation on Probability of Winning

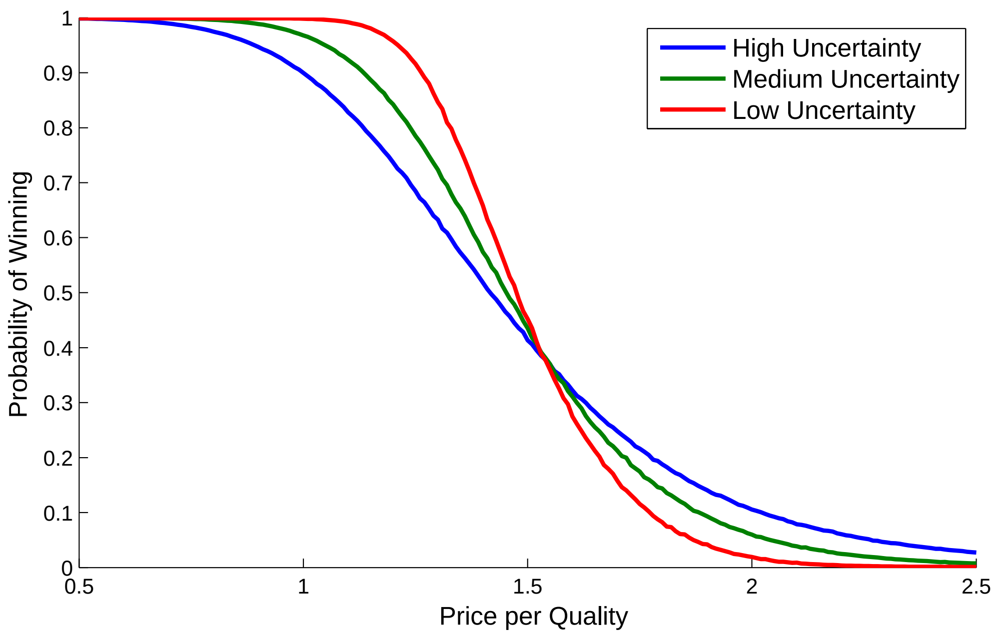
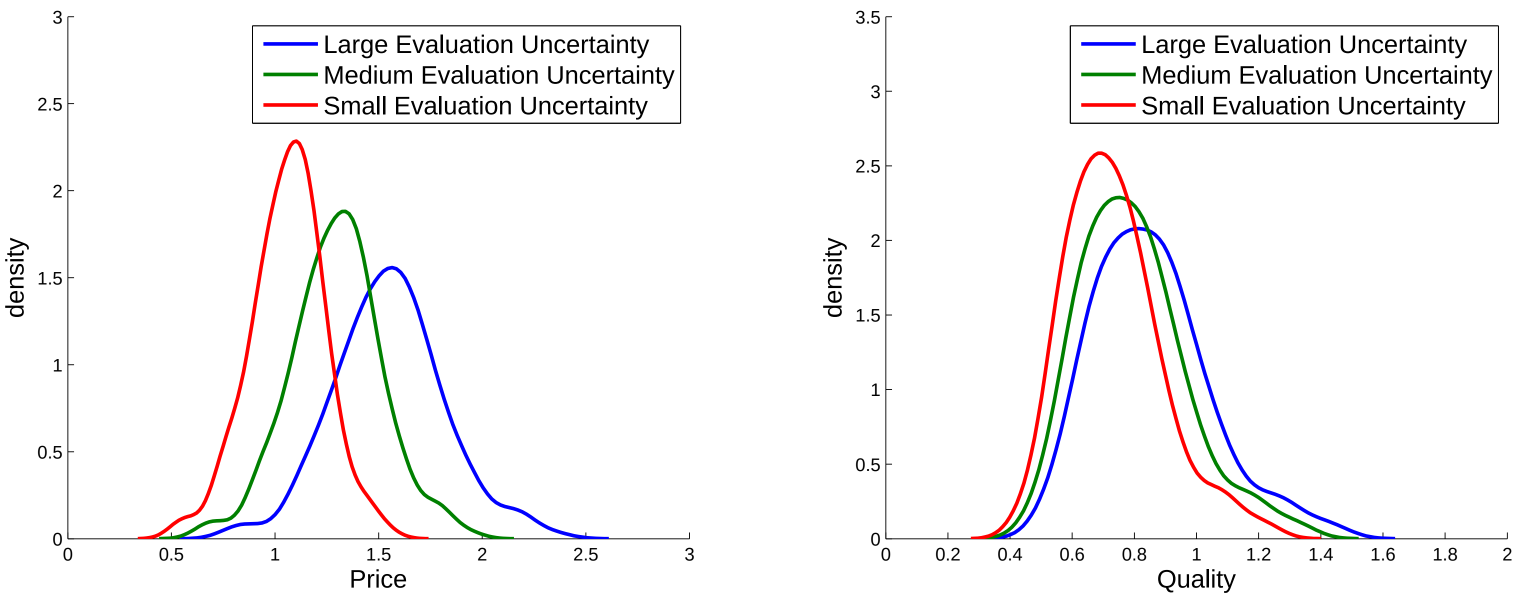


Figure 3: The Effects of Uncertain Evaluations on Equilibrium Bidding Strategies



4 Identification

Identification of the model is challenging due to unobserved design quality choices. More specifically, the econometrician does not observe design quality, but only observes some noisy signals of quality. As design quality is unobserved, the probability of winning function G_i and its density g_i cannot be directly evaluated from the data, precluding the standard inversion approach pioneered by Guerre, Perrigne, and Vuong (2000).[21] The identification problem is made more complex due to unobserved auction and reviewer heterogeneities, which may be confounded with the degree of evaluation uncertainty. Further, the set of bidders and reviewers are chosen endogenously by the auctioneer. Therefore, a plausible identification strategy needs to deal with selection problem.

Public procurement auctions of infrastructure projects involve a substantial amount of unobserved auction heterogeneity: auction heterogeneity observed by all participating bidders, but unobserved by the econometrician. As demonstrated by Krasnokutskaya (2011), ignoring the presence of unobserved auction heterogeneity exaggerates the dispersion of bids related to bidder heterogeneity. The exaggerated dispersion of bids in turn underestimates the impact of evaluation uncertainty on bidding strategies since evaluation noises may swap bidders' rankings only if bidders are in a close competition, and bidders are likely to be in a close competition when bidders' types are densely distributed.

Unobserved reviewer heterogeneity also plays a crucial role in identifying the degree of evaluation uncertainty. Reviewers may differ in their evaluation standards, and a lenient reviewer may assign a high score to every design, which exerts little effect in determining the rankings of bidders. The econometrician would exaggerate the degree of evaluation uncertainty if unobserved reviewer heterogeneity is ignored, confounding with idiosyncratic evaluation noise that can swap the rankings of design proposals.

Selection of bidders and reviewers by the auctioneer is not a random process in a DB auction. If the auctioneer is aware of the effect of evaluation uncertainty on bidding strategies, then the auctioneer may tend to let more reviewers and bidders into an auction for a complex project in order to mitigate the effect of evaluation uncertainty on auction outcomes. Such selection process would imply that bidders are inefficient on average in a project with a larger number of bidders and reviewers relative to a project with a fewer number of bidders and reviewers. To address this selection issue, I allow for the distribution of bidders' private information to depend on the number of bidders and reviewers in an auction as in Campo *et al.* (2011). As

[21] Estimation of design quality by the average reviewers' evaluation faces an incidental parameter problem, since each auction involves only a few reviewers.

the model primitives are not identified without specifying a functional form for $C(.)$, I assume a simple power function for $C(.)$ for the rest of the paper.[22]

Identifying Assumption 1. $C(.)$ *is a power function:*

$$C(q) = q^{\gamma},$$

where $\gamma > 1$ *for convexity.*

The rest of the section is organized as follows. First, I describe the information structure of bidders: what bidders know about a project and their rivals at the time of bidding. Second, I introduce unobserved reviewer heterogeneity and evaluation noise into the model without uncertain evaluations. Lastly, I derive reduced form expressions that characterize the equilibrium bidding behavior, and show how to identify the primitives of the model semiparametrically.

4.1 Information structure on bidders' types

This subsection specifies the information structure, describing what bidders observe at the time of bidding. The econometrician has an access to a vector of exogenous auction characteristics, Z_a, which may include the engineer's estimate of the project cost and project types (e.g., road, bridge, building construction, etc). While the model in Section 3 abstracts from information asymmetry between bidders and the econometrician, empirical results can be significantly influenced by an assumption on the information structure. I.A.2 below makes explicit who observes which component of a particular bidder's cost.

Identifying Assumption 2. *Bidders' types are given by:*

$$vc_{ia} = \exp\{Z_a\beta_v + \theta_a^v + \varepsilon_{ia}^v\} \tag{6}$$

$$e_{ia} = \exp\{Z_a\beta_e + \theta_a^e + \varepsilon_{ia}^e\}, \tag{7}$$

where Z_a is a vector of observed characteristics known to bidders and also to the econometrician. θ_a^v and θ_a^e are both unobserved auction heterogeneity: observed by all participating bidders but unobserved by the econometrician. θ_a^v and θ_a^e are assumed to be independent of Z_a, ε_{ia}^v and ε_{ia}^e. ε_{ia}^v and ε_{ia}^e are both private information observed only by bidder i, which are independently and

[22] $C(.)$ can be identified from the data if both $C(.)$ and the distribution functions of bidders' private information are parametrized as in Campo *et al.* (2011). Here, I assume that the cost function is known to the econometrician, and identify the distribution of primitives nonparametrically.

identically distributed. I.A.2 implies multiplicative separability of bidding strategies in each cost component as shown in the following proposition.

Proposition 5. *Let* $\{\boldsymbol{q(Z, \theta, \varepsilon^v, \varepsilon^e)},\ \boldsymbol{b(Z, \theta, \varepsilon^e)}\}$ *denote an equilibrium strategy profile of bidders. Consider a monotone pure strategy equilibrium characterized by a set of first-order conditions (4) and (5). Then, the equilibrium reduced form PQR strategies are additively log-separable in every cost component. Further, structural equation (4) implies that pricing strategies can also be expressed as a function of the decomposed PQR strategies:*

$$\dot{b}(Z_a, \theta_a, \varepsilon^e_{ia}) \;=\; Z_a\tilde{\beta}_e + (1/\gamma_0)\,\theta^e_a + \dot{s}_{ia} \tag{8}$$

$$\dot{q}(Z_a, \theta_a, \varepsilon^v_{ia}, \varepsilon^e_{ia}) \;=\; Z_a\tilde{\beta}_v + (1/\gamma)\,\theta^e_a + \gamma_1\,(\dot{s}_{ia} - \theta^v_a - \varepsilon^v_{ia}), \tag{9}$$

$\forall\ i \in \mathcal{N}$ *where* $\dot{x} \equiv ln(x)$ *for all variable* x, $\dot{s}_{ia} \equiv \dot{b}(0, 0, \varepsilon^e_{ia})$, $\gamma_0 \equiv \gamma/(\gamma - 1)$, $\gamma_1 \equiv 1/(\gamma - 1)$, $\tilde{\beta}_e = (1/\gamma_0)\,\beta_e$, *and* $\tilde{\beta}_v \equiv (1/\gamma)\,\beta_e - \gamma_1\,\beta_v$.

Proof in Appendix. Proposition 5 shows that bidding strategies are decomposed into a sole function of ε^e_{ia} with the remainder appearing in the form of the primitives, which significantly simplifies the identification problem. As the design quality is unobserved by the econometrician, I provide the link between reviewers' evaluations and design quality next.

4.2 Unobserved reviewer heterogeneity and evaluation noise

Recall that, instead of design quality q_{ia}, the econometrician observes each reviewer's evaluation of each bidder's design proposal, $\{q^0_{ria} : r = 1, 2, ..., R_a\}$, where r is the reviewer subindex, and R_a is the number of reviewers in auction a.

Reviewer heterogeneity is decomposed into three components: measure heterogeneity η_a, unobserved reviewer heterogeneity μ_{ra}, and evaluation noise ξ_{ria}. Measure heterogeneity may capture heterogeneity in scoring difficulty: some auctions may be more difficult for bidders to score high than other auctions. Unobserved reviewer heterogeneity captures standard differences across reviewers, such as reviewers' leniency in design evaluations. Lastly, evaluation noise captures idiosyncratic evaluations of design proposals. I.A.3 describes the link between design quality and reviewers' evaluations.

Identifying Assumption 3. *Reviewer r's evaluation of bidder i's design,* q^0_{ria}, *is noisy but an unbiased estimate of true quality* q_{ia}*:*

$$\dot{q}^0_{ria} \;=\; \dot{q}_{ia} + \dot{w}_{ria},$$

where $\dot{w}_{ria} \equiv \eta_a + \mu_{ra} + \xi_{ria}$. Note that η_a is another form of unobserved auction heterogeneity, which has nothing to do with bidders' costs. Both measure heterogeneity and unobserved reviewer heterogeneity capture design rank preserving variation in evaluations. That is, a swap in design rankings does not occur for these two types of heterogeneities since realizations of these random variables affect evaluations of different design proposals equally.

With I.A.3, equation (8) and (9) can be rewritten in terms of price per reviewer's evaluation and reviewer's evaluation, which is observed to the econometrician.

$$\begin{aligned} \dot{\varphi}_{ria} &= Z_a \tilde{\beta}_f + (1/\gamma_0)\, \theta^e_a + \dot{s}_{ia} + \dot{w}_{ria} && (10) \\ \dot{q}^0_{ria} &= Z_a \tilde{\beta}_v + (1/\gamma)\, \theta^e_a + \gamma_1 \left(\dot{s}_{ia} - \theta^v_a - \varepsilon^v_{ia}\right) + \dot{w}_{ria}, && (11) \end{aligned}$$

where $\varphi_{ria} \equiv p_{ia}/q^0_{ria}$ (i.e., price per reviewer r's evaluation). I define (10) and (11) as a Reduced Form Factor (RFF) model since the two equations are represented as functions of the primitives of the model, and are linked through the latent factors. Note that additive separability of $\dot{w}_{ria}$ also follows from the model, and ξ_{ria} does not enter bidders' states as bidders do not know how reviewers evaluate designs ex-ante.[23]

Lastly, I state a technical assumption which guarantees nonparametric identification of the distribution functions of the random components in (10) and (11).

Identifying Assumption 4. *The probability density functions of the individual random components, $f_x\ \forall\, x$, are continuously differentiable and strictly positive on the interior of $(\underline{x}, \bar{x})$.*

Given I.A.1-I.A.4, the primitives of the model to be identified are: (i) the distribution of unobserved reviewer heterogeneity F_μ, (ii) the distribution of evaluation noise F_ξ, and (iii) the distribution of variable cost and efficiency type, F_v and F_e, respectively.[24] The distributions of F_s, F_v, F_μ, and F_ξ are nonparametrically identified from the RFF model. To identify F_e, I first obtain the marginal distribution of s_{ia}, F_s, from the RFF model. Then, I recover F_e from the first order condition (5) and the set of distributions $\{F_s,\ F_v,\ F_\mu,\ F_\xi\}$. I elaborate on the identification of the model below.

[23] The identification arguement here does not rely on whether η_a and μ_{ra} are observed to bidders or not.

[24] The joint distribution of unobserved auction heterogeneities is neither identified nor the focus of the analysis here. The idea is to net out all sorts of unobserved auction heterogeneities to exploit within-auction variation in bids.

4.3 Semiparametric identification of the RFF model

In order to identify the degree of evaluation uncertainty from a bidder's point of view, I isolate the part of a reviewer's evaluation that bidders know at the time of bidding from the part they do not. The RFF model exploits the fact that bidders do not observe reviewers' evaluations of their designs at the time of bidding, and also reviewers do not observe bidders' price bids at the time of evaluation. In particular, the RFF model assumes that disagreement among reviewers on the design quality of a proposal is unknown to the bidder, but the part of design quality that is agreed among reviewers is known to the bidder at the time of bidding.

Note that the notion of design quality here is broad in the sense that the latent factor captures all the information that reviewers have about bidder i at the time of evaluation. That is, if all reviewers agree that a particular design proposal is of high quality, then the design is deemed to be of high quality. Therefore, a bidder's reputation, or the impression that reviewers receive from a particular bidder in the pre-proposal meeting, can be regarded as a part of design quality as long as reviewers agree and a bidder knows what reviewers know about themselves.

Proposition 6. *Given I.A.1-I.A.4, $\{F_\mu, F_\xi, F_s, F_v, F_e\}$ are all nonparametrically identified.*

For simplicity, I omit the observables, Z_a, in showing the nonparametric identification of the RFF model below.[25] The identification argument here closely follows Carneiro, Hansen, and Heckman (2003).

First, the measure of evaluation uncertainty is identified by exploiting the fact that multiple reviewers evaluate multiple designs in an auction. Intuitively, the identification of evaluation noise comes from a discrepancy in reviewers' evaluations net of reviewer specific characteristics (e.g., leniency):

$$(\dot{\varphi}_{ria} - \dot{\varphi}_{r'ia}) = (\dot{s}_{ia} - \dot{s}_{i'a}) + (\xi_{ria} - \xi_{r'i'a}) \quad \text{for } i' \neq i. \tag{12}$$

As the left hand side of (12) is observed, and the right hand side is a sum of *i.i.d.* random variables, F_s and F_ξ are both nonparametrically identified by deconvolution. Similarly,

$$(\dot{\varphi}_{ria} - \dot{\varphi}_{r'ia}) \;=\; (\mu_{ra} - \mu_{r'a}) + (\xi_{ria} - \xi_{r'ia}) \;\text{ for } r' \neq r. \tag{13}$$

Since the distribution of the LHS of (13) is known, F_μ can also be identified by deconvolution.

[25] The proof trivially goes through with the observables included.

Second, bidder heterogeneities can also be identified by taking into account the correlation between ε_{ia}^{v} and ε_{ia}^{e}. As γ is known and $\dot{w}_{ria} - \dot{w}_{ri'a} = \xi_{ria} - \xi_{r'ia}$, it follows that:

$$(\dot{q}_{ria}^{0} - \dot{q}_{ri'a}^{0}) - \gamma_1 (\dot{\varphi}_{ria} - \dot{\varphi}_{ri'a}) \quad = \quad \gamma_1 (\varepsilon_{ia}^{v} - \varepsilon_{i'a}^{v}) + (1-\gamma_1)(\xi_{ria} - \xi_{r'ia}) \quad \text{for } i' \neq i,$$

and so the marginal distribution of ε_{ia}^{v}, F_v, is identified again by deconvolution. This completes the identification of the RFF model.

Finally, the distribution of efficiency private information F_e can be identified using equation (5) by integrating over F_s, F_μ, and F_ξ, as in Guerre, Perrigne and Vuong (2000), such that:

$$\kappa(\gamma) \left(s_{ia}^{\gamma_0} + \gamma_0 \, s_{ia}^{\gamma_1} \frac{G_{ia}(s_{ia}, \boldsymbol{\psi_{-ia}})}{g_{ia}(s_{ia}, \boldsymbol{\psi_{-ia}})} \right) \quad = \quad \exp\{\varepsilon_{ia}^{e}\}, \tag{14}$$

where $\kappa(\gamma) \equiv \gamma^{-\gamma_1} - \gamma^{-\gamma_0}$. Thus, F_e can be identified from repeated auctions.

There are several remarks to be made. First, the distribution functions F_e and F_v depend on the number of bidders and reviewers as the probability of winning function G_{ia} and g_{ia} depends on the number of bidders and reviewers. I denote the distribution of bidders' efficiency private information by $F_e(.; N, R)$ to make explicit the dependency of the distribution on the number of bidders and reviewers. Second, the identification strategy deployed here does not require any of the unobserved auction heterogeneity to be identified as all the unobserved auction heterogeneities are differenced out. Lastly, F_s, F_μ, and F_ξ are all identified without the knowledge of γ while F_v cannot be recovered without the knowledge of γ. As the equilibrium price and quality choices of bidders in a counterfactual experiment can be significantly influenced by an assumed value of γ, I present estimates and conduct counterfactual experiments with varying values of γ.

5 Estimation

While the estimation steps closely follow the identification argument, data specific issues need to be considered. In particular, estimating the distributions of the primitives for all possible combinations of the number of bidders and reviewers places a burden on the small sample.

To deal with this issue, I assume a single equilibrium bid distribution in the data, and recover the distributions of the primitives that rationalize the observed bids' distributions. While the assumption of single bid distribution may be strong, I show that this assumption is consistent with the observed distribution of bids.

I further impose joint normality assumption on the distributions of $\dot{s}_{ia}$ and ε_{ia}^{v} given the small sample. This parametrization indeed allows for potential corelation between ε_{ia}^{v} and ε_{ia}^{e} in a parsimoneous manner.

I estimate the parameters of the RFF model by Method of Moments for varying parameter values for γ, and then recover the distribution of efficiency private information F_e via simulation of the RFF model combined with the first order optimality condition in (14).

5.1 Estimation steps

Step 1: A method of moments estimator for the RFF model.
Let σ_j denote the variance of unobservable $j \in \{s,\, \mu,\, \xi,\, v,\, e\}$, and denote the within-bidder type covariance by $\delta \equiv Cov(\varepsilon_{ia}^{v},\, \dot{s}_{ia})$. First, I estimate the equilibrium bidding strategies in (10) and (11) by OLS, partialling out the effects of observables. Then, I obtain OLS residuals (denoted by $\hat{q}_{ria}$ and $\hat{\varphi}_{ria}$) and estimate the variance components by the sample variance covariances specified below:

$$\begin{aligned}
\hat{\sigma}_{\xi} &= M[\hat{q}_{ria}^{2}] - M[\hat{q}_{ria}\hat{q}_{ri'a}] + M[\hat{q}_{ria}\hat{q}_{r'i'a}] - M[\hat{q}_{ria}\hat{q}_{r'ia}] \\
\hat{\sigma}_{s} &= M[\hat{\varphi}_{ria}\hat{\varphi}_{r'ia}] - M[\hat{\varphi}_{ria}\hat{\varphi}_{r'i'a}] \\
\hat{\sigma}_{\mu} &= M[\hat{q}_{ria}\hat{q}_{ri'a}] - M[\hat{q}_{ria}\hat{q}_{r'i'a}] \\
\hat{\delta} &= M[\hat{\varphi}_{ria}\hat{\varphi}_{r'ia}] - M[\hat{\varphi}_{ria}\hat{\varphi}_{r'i'a}] - (M[\hat{\varphi}_{ria}\hat{q}_{r'ia}] - M[\hat{\varphi}_{ria}\hat{q}_{r'i'a}])/\gamma_{1} \\
\hat{\sigma}_{v} &= (M[\hat{q}_{ria}\hat{q}_{r'ia}] - M[\hat{q}_{ria}\hat{q}_{r'i'a}])/\gamma_{1}^{2} - \hat{\sigma}_{s} + 2\,\hat{\delta}
\end{aligned} \tag{15}$$

where $M[.]$ denotes sample mean.
Step 2: A simulated estimator for $F_e(.;N,R)$.
The second step of the estimation procedure involves simulation. (i) Draw a pseudo random variable s_{1k} from $\hat{F}_s$, (ii) numerically integrate over the distribution of rivals' strategies $\boldsymbol{s_{-1l}} = [s_{2kl}, s_{3kl}, ..., s_{Nkl}]$ and evaluation noise $\boldsymbol{w_l} = [w_{1k1}, w_{2kl}, ..., w_{Nkl}]$ by repeatedly drawing from $\boldsymbol{\hat{F}_{-s}}$ and $\boldsymbol{\hat{F}_{w}}$ where I set $L = 10^3$ to obtain the estimate of the probability of winning for a given number of bidders N and reviewers R by:

$$\hat{G}(s_{1k};N,R) \;\; = \;\; \int\!\!\int \mathbf{1}\{\dot{s}_{1k} - \dot{w}_{1kl} < \dot{s}_{jkl} - \dot{w}_{jkl} \;\text{ for } j \neq 1 \in \mathcal{N}\} d\boldsymbol{\hat{F}_{-s}} d\boldsymbol{\hat{F}_{w}}.$$

Similarly, $\hat{g}(s_{1k};N,R)$ can be obtained by numerically differentiating $\hat{G}(s_{1k};N,R)$. Note here that $\hat{G}(s_{1k};N,R)$ is obtained via simulation, and N and R are not taken from the data. (iii) I obtain the simulated efficiency private cost, $\hat{\varepsilon}^e_{1k}$, by evaluating equation (14).

$$\kappa(\gamma) \left(s_{1k}^{\gamma_0} + \gamma_0 \, s_{1k}^{\gamma_1} \frac{\hat{G}_{ia}(s_{1k}; N, R)}{\hat{g}_{ia}(s_{1k}; N, R)} \right) \quad = \quad \exp\{\hat{\varepsilon}^e_{1k}\}. \tag{16}$$

Iterate (i) through (iii) K times (where I set $K = 10^3$) to estimate the distribution of $\hat{\varepsilon}^e_{1k}$ by repeatedly evaluating (16). I compute $\hat{F}_e(.; N, R)$, which is the distribution of $\hat{\varepsilon}^e_{1k}$ for all possible combinations of hypothetical number of bidders and reviewers predicted by the model.

5.2 Estimation results

Table 6 shows estimates of variance components for varying level of γ. The vector of observed auction characteristics includes an engineer's estimate of project cost, the number of bidders and reviewers in the auction to capture the effect of competition and evaluation uncertainty. Project type dummies control for observed auction heterogeneity.[26]

The first significant finding is the large estimate of evaluation uncertainty σ_ξ. Evaluation uncertainty is as large as 39% of the within-auction heterogeneity in PQR bids σ_s. While not shown in Table 6, σ_s would be significantly overestimated if unobserved auction heterogeneities are not taken into account. A consistent estimation of σ_s is particularly relevant in the context of the analysis here since evaluation noise is likely to swap the ranking of bidders only in a close competition. Therefore, overestimating the dispersion in private information of bidders results in an underestimation of the effect of evaluation uncertainty on bidders' behavior.

Another important observation here is the close positive relation between the values of γ and the estimates of σ_v. Intuitively, a large variation in within-reviewer variation in evaluations across bidders can be explained either by (i) a large variation in vc_i: a large variation in design quality resulting from bidder heterogeneity in the cost of design or (ii) a small value of γ: a large variation in design quality resulting from a low elasticity of substitution between price and design quality. Therefore, the estimate of σ_v is increasing in γ to account for the extent of variation in design quality present in the data.

What seems puzzling at first glance here is the insignificant coefficient estimate on the number of bidders. If competition is all it captures, the insignificant estimate is intriguing. However, the number of bidders may be correlated with the distribution of bidders' private information. For example, if the FDOT observes project complexity, and if the FDOT tends to

[26]While the model does not predict that bidders' strategies are additively log-linear in bidder level characteristics, I control them to approximate observed bidder heterogeneity when estimated. Project types are classified into road, bridge, building, and others.

allow more applicants to participate in an auction for a more complex project, then the effect of competition on pricing and design decisions can be completely offset by project complexity. That is, bidders are on average inefficient at implementing a complex project, and therefore the effect of competition on bidding strategies is hidden by project complexity. Thus, the coefficient estimate on the number of bidders captures the total effect of competition and FDOT's selection.[27] For the same reason, correlation between the number of reviewers and bidders' behavior can be hidden by project complexity. Note that the estimation result is consistent with the assumption of single equilibrium bid distribution.

Figure 4 shows the distribution of bidders' private cost information for varying numbers of bidders and reviewers. Intense competition and a large evaluation uncertainty are associated with a right shift of the distribution of private cost information. That is, the more bidders or more reviewers there are, the more inefficient each bidder is on average. This finding is in line with the estimation result obtained from the RFF model. After computing the equilibrium using the estimated distribution of efficiency level, I find no significant difference in mean bids across the number of bidders. That is, the competition effects on pricing and design strategies are offset by the asymmetry in the distribution of private information. Therefore, the structural model here is consistent with the fact that the number of bidders and the number of reviewers are insignificantly correlated with both pricing and design strategies in the RFF model.

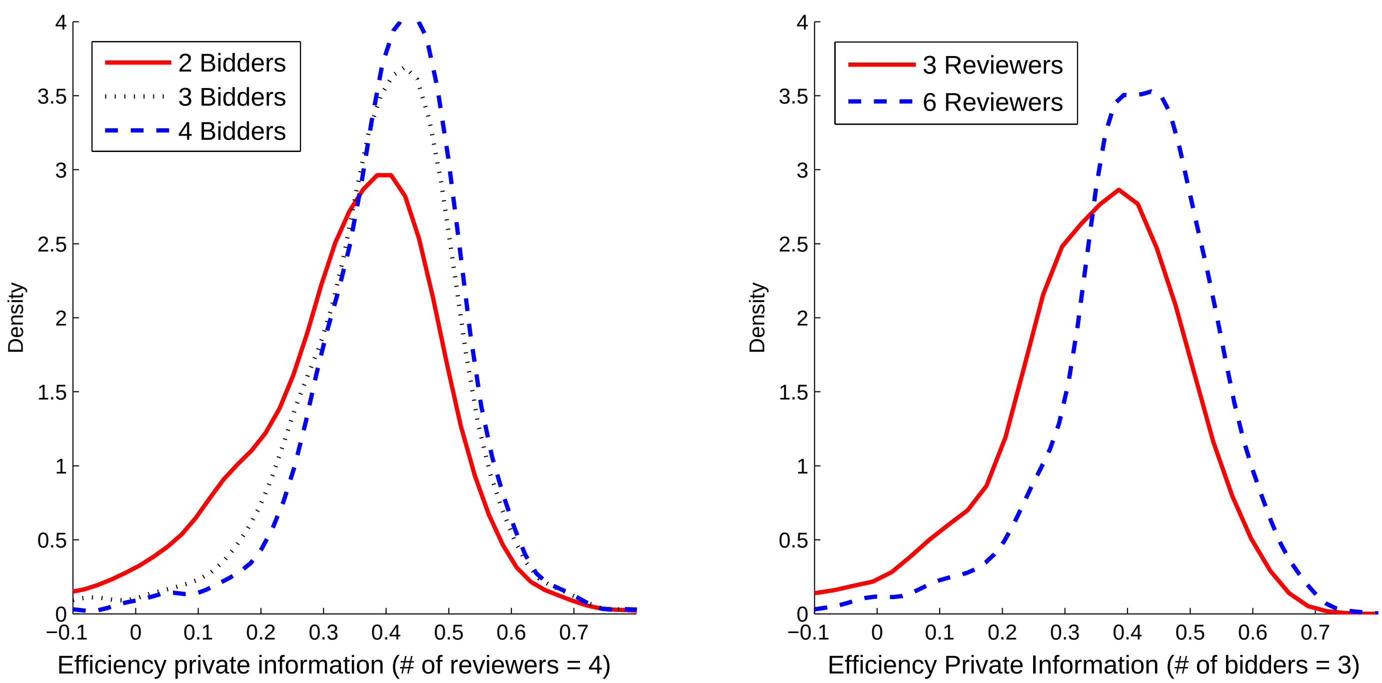
[27] Another potential explanation for the statistically insignificant correlation between the number of bidders and bids can be found in Somaini (2011) where it is shown that competition does not necessarily induce more aggressive bidding in the presence of common value signals. While Somaini (2011) shows an interesting insight, I abstract from common value aspect of procurement auction in this paper.

Table 6: Reduced Form Factor Model: Estimation Results

	(i) $\gamma = 4$		(ii) $\gamma = 8$		(iii) $\gamma = 12$	
	$\dot{\varphi}_{ria}$	$\dot{q}^0_{ria}$	$\dot{\varphi}_{ria}$	$\dot{q}^0_{ria}$	$\dot{\varphi}_{ria}$	$\dot{q}^0_{ria}$
# of Bidders / Auction	.0416	-.00637	.0416	-.00637	.0416	-.00637
	(.0566)	(.0127)	(.0566)	(.0127)	(.0566)	(.0127)
# of Reviewers / Auction	.0466	.00077	.0466	.00077	.0466	.00077
	(.0411)	(.00838)	(.0411)	(.00838)	(.0411)	(.00838)
σ_s , σ_v	.0148	.0443	.0148	.137	.0148	.289
	(.00163)	(.00537)	(.00163)	(.0194)	(.00163)	(.0433)
σ_ξ , σ_μ	.00583	.00195	.00583	.00195	.00583	.00195
	(.000487)	(.000308)	(.000487)	(.000308)	(.000487)	(.000308)
δ	.0213		.0215		.0305	
	(.00485)		(.00240)		(.00391)	
Project Type Fixed Effecs	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effecs	Yes	Yes	Yes	Yes	Yes	Yes
Obs	1296	1296	1296	1296	1296	1296

Reduced form factor model is estimated for varying values of γ. Bootstrapped standard errors in parentheses. Price per reviewer's evaluation φ_{ria} is normalized by engineer's cost estimate to control for project size heterogeneity. Engineer's cost estimate is an estimate of winning price predicted by an FDOT engineer prior to an auction. Project type is assigned to each project based on project description on bid tabs. Every project is classified and assigned one of road, bridge, building, mixed project, and other dummies.

Figure 4: Distribution of Efficiency Private Information



6 Does evaluation uncertainty matter?

As shown in Section 3, an increase in evaluation uncertainty leads to an increase in the expected price of design quality, and a greater dispersion in both price and design quality. The above behavioral effects coupled with misallocations of project awards due to subjective evaluations may have significant adverse effects on the auctioneer.

This section quantifies the impact of evaluation uncertainty through a change in the number of reviewers. More specifically, I examine to what extent evaluation uncertainty can be mitigated by adding more reviewers to the evaluation process. As demonstrated in the following, the marginal effect of additional reviewers dissipates quickly since evaluation uncertainty is convex in the number of reviewers. Moreover, allocating a large number of reviewers to a review task may come with a large opportunity cost to the FDOT, which is not captured in the model. This counter factual exercise may also suffer from potential multiplicity of equilibria since the model does not guarantee uniqueness of equilibria. Therefore, a change in a parameter of the model could lead to a shift in equilibrium.

To shut down the effect of evaluation uncertainty on bidders' behavior without additional reviewers, I propose a simple auction mechanism, defined as PQR auction. The alternative auction format keeps the setting of a DB auction except that every bidder submits a price per unit of design score rather than a price for an entire project. As the allocation of a project is not affected by the subjective evaluations of reviewers, it restores the efficient allocation of a project: the most efficient bidder among the set of bidders always wins. Moreover, its unique equilibrium bidding strategy corresponds to the equilibrium bidding strategy of a DB auction with no evaluation uncertainty. Thus, a switch from the DB auction rule to the PQR auction rule necessarily lowers the expected price of design quality, and also reduces the dispersion in price and design quality.

6.1 Simulation of DB auction with varying number of reviewers

Consider a symmetric average DB auction with three bidders and a varying number of reviewers R. The degree of evaluation uncertainty is $\hat{\tau}(R)$, which is a decreasing and convex function of R. The simulation results are shown in Table 7.

Table 7: Distributions of Auction Outcomes at Various Number of Reviewers

	Mean		Standard Deviation	
$\gamma = 4$	Winning Price	Winning Quality	Winning Price	Winning Quality
$R = 1$	20.3	84.0	2.52	6.33
$R = 2$	19.7	83.5	2.37	6.27
$R = 3$	19.5	83.3	2.32	6.25

Each row corresponds to a hypothetical number of reviewers, which alters the degree of evaluation uncertainty. The mean and standard deviation of winning price and design quality are computed through model simulation with the estimates obtained in the previous section. The counterfactual exercise is conducted using an average project in the sample. Winning prices are expressed in $1,000,000 while winning quality scores are out of 100 points.

Adding a reviewer to a DB auction with a reviewer would reduce the winning price by 1.3%, and the winning design quality by 0.25% on average. The effects of additional reviewers on the dispersion of auction outcomes are somewhat larger than those on its expected value. The standard deviations of the winning price and design quality decline by 2.66% and 0.41%, respectively.

While an increased number of reviewers mitigates evaluation uncertainty, the marginal effect of an additional reviewer declines quickly due to the convexity of evaluation uncertainty in the number of reviewers. In addition, assigning many employees of the FDOT to the review task can be prohibitively costly. To mitigate evaluation uncertainty without incurring additional administrative cost of appointing reviewers, I propose an alternative auction mechanism that shuts down the incentive as well as allocational effects of evaluation uncertainty on auction outcomes.

6.2 PQR auction with design score contingent transfer

Under this alternative auction format, bidders submit a price of design quality (PQR) and a design proposal. More specifically, I consider an auction in which the winner is selected based solely on the lowest announced PQR, and the contractor receives the product of its own PQR bid and design quality score. All other procedures remain exactly the same as in a DB auction. This auction mechanism is of interest as it shuts down the effect of evaluation uncertainty on bidding strategies without reducing the amount of evaluation uncertainty. Neither a bidder's chance of winning nor its ex-post payoff is affected by uncertain design evaluations of its rivals, and thus bidders do not respond to the uncertainty in rivals' design evaluations.[28]

[28] Bidders would also be non-responsive to uncertainty in its own design evaluations under risk neutrality.

Bidders simultaneously choose both a PQR, b_i, and a design quality, q_i. The winner is selected by the lowest b_i. Design quality score is determined by the average across reviewers' evaluations, $q_i w_i$. Non-winning bidders receive a zero transfer. Let $E_w[.]$ denote the expectation operator over the distribution of w_i. Also, denote the probability of winning function conditional on bidder i's own price bid and rivals' PQR strategies, $\boldsymbol{\psi_{-i}}$, by $Pr(\text{bidder } i \text{ wins}|b_i, \boldsymbol{\psi_{-i}})$. Note that the probability of winning function is independent of the distribution of evaluation noise F_w. Then, the interim expected payoff of bidder i is defined as:

$$\pi_i^{int} = \max_{b_i, q_i} E_w[Pr(\text{bidder } i \text{ wins}|b_i, \boldsymbol{\psi_{-i}})(b_i q_i w_i - vc_i\, C(q_i) - fc_i)].$$

Unbiased evaluation noise with risk neutrality assumption gives:

$$\pi_i^{int} = \max_{b_i, q_i} Pr(\text{bidder } i \text{ wins}|b_i, \boldsymbol{\psi_{-i}})(b_i q_i - vc_i\, C(q_i) - fc_i).$$

Therefore, the model is identical to that of a DB auction without evaluation noise where $p_i = b_i q_i$. The first-order condition with respect to q_i gives:

$$b_i = vc_i\, C_q(q_i), \tag{17}$$

which is exactly the same condition as the ratio of first-order conditions for the case of a DB auction. Therefore, bidder i's problem can again be reduced to a one-dimensional choice problem, and its PQR strategy is again a function of single-index e_i, since:

$$\begin{aligned}\pi_i^{int} &= \max_{b_i} Pr(\text{bidder } i \text{ wins}|b_i, \boldsymbol{\psi_{-i}})(b_i q_i(b_i) - vc_i\, C(q_i(b_i)) - fc_i) \\ &= \max_{b_i} Pr(\text{bidder } i \text{ wins}|b_i, \boldsymbol{\psi_{-i}})\,(u(b_i) - e_i).\end{aligned}$$

It follows that, for a symmetric game, a unique symmetric equilibrium exists where the monotone PQR strategy $\psi(e_i)$ is strictly increasing and differentiable in e_i, such that:

$$\psi(e_i) = u^{-1}\left(e_i + \frac{\int_{e_i}^{\bar{e}}(1 - F_e(x))^{N-1}dx}{(1 - F_e(e_i))^{N-1}}\right), \tag{18}$$

and corresponding pricing strategy $p(e_i, vc_i)$, and design strategy $q(e_i, vc_i)$ are determined by:

$$\begin{aligned} q(e_i, vc_i) &= C_q^{-1}\left(\frac{\psi(e_i)}{vc_i}\right) \\ p(e_i, vc_i) &= \psi(e_i)\, q(e_i, vc_i). \end{aligned}$$

Proposition 7. *There exists a unique symmetric Bayesian Nash equilibrium of PQR auction in which PQR strategy, $\psi(e_i)$, is strictly increasing and differentiable.*

Proof in Appendix. I simulate and obtain the distribution of winning price and design quality in this alternative auction. Table 8 compares auction outcomes from DB and PQR auction.

Table 8: Comparison of DB and PQR auction outcomes with varying values of γ

	Mean		Standard Deviation	
$\gamma = 4$	Winning Price	Winning Quality	Winning Price	Winning Quality
DB auction	20.3	84.0	2.52	6.33
PQR auction	19.0	82.7	2.24	6.23
Percentage difference	6.76%	1.58%	11.5%	1.50%

	Mean		Standard Deviation	
$\gamma = 8$	Winning Price	Winning Quality	Winning Price	Winning Quality
DB auction	20.1	84.1	1.92	4.47
PQR auction	19.0	83.6	1.67	4.46
Percentage difference	5.67%	0.59%	13.8%	0.23%

	Mean		Standard Deviation	
$\gamma = 12$	Winning Price	Winning Quality	Winning Price	Winning Quality
DB auction	20.1	83.5	2.04	4.31
PQR auction	19.0	83.4	1.83	4.31
Percentage difference	5.45%	0.45%	10.6%	0.13%

The mean and standard deviation of winning price and design quality are computed for DB and PQR auction through model simulation with the estimates obtained in the previous section.The counterfactual exercise is conducted using the average road project in the sample. $R = 1$ and $N = 3$. Winning prices are expressed in \$1,000,000 while winning quality scores are out of 100 points.

The PQR auction restores the DB auction outcomes with no evaluation uncertainty. Upon a switch from the DB auction rule to the PQR auction rule, the expected winning price and design quality decline by 6.76% and 1.58%, respectively. The standard deviation of winning price and design quality decline by 11.5% and 1.50%, respectively.

The above analysis has several economic implications. To avoid the adverse effects of a subjective client's evaluation on contracting outcomes, the client may want to adopt an objective winner selection rule. In many cases, this criteria is some function of a price. An interesting finding of the paper is that not all of the design criteria need to be objectively measurable, and no uncertainty equilibrium can be implemented without reducing uncertainty itself.

It is important to note, however, that the result obtained in this counterfactual experiment exploits the risk neutrality assumption. If bidders are risk averse, the proposed PQR auction is not equivalent to a DB auction with no evaluation uncertainty. Moreover, the alternative auction format may induce reviewers to lower design scores if the reviewers wish to reduce the procurement cost. Even without such reviewers' incentive to lower the cost, the auctioneer needs to be careful in netting out unobserved reviewer heterogeneity since bidders could lose profit from simply facing stringent reviewers in the alternative auction.

7 Conclusion

This paper studies the effects of uncertain design evaluations on competing suppliers' behavior using a sample of Design-Build auctions from the Florida Department of Transportation. I document the presence of evaluation uncertainty, which affects the rankings of bidders through introducing *luck* into the winner selection process. A structural model that incorporates uncertainty in both design evaluations and rivals' bids is developed and estimated, taking into account potentially confounding heterogeneities, such as unobserved auction heterogeneity and unobserved reviewer heterogeneity. The structural approach is consistent with the observed fact that both the number of bidders and reviewers have no effect on bidders' behavior on the surface. The paper also provides the first attempt in the literature on multi-attribute auctions to estimate structural parameters when some attributes of a bid are unobserved by the econometrician. The economic significance of evaluation uncertainty is demonstrated through simulation exercises. An increase in evaluation uncertainty not only generates a higher winning price per unit of design quality on average, but also exacerbates dispersion in auction outcomes, resulting in greater uncertainty in auction outcomes from the auctioneer's standpoint. A simple adjustment in the auction rule may completely shut down the impact of uncertain evaluations on bidding strategy. Further, the adjustment in the auction rule precludes an inefficient allocation of a project by selecting the most efficient bidder.

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8 Appendix (for online publication)

8.1 Definition of observables

- Engineer's estimate of project cost: A proxy for the project size.
- Distance: Distance between project site and the closest branch of bidder.
- Utilization Rate: A bidder's backlog per capacity. Backlog is defined as the total dollar value of projects ongoing at the time of bidding. Capacity of a bidder is defined as the maximum backlog during the period the sample is taken from. Backlog and capacity are calculated using all other types of auctions and DB auctions procured by the FDOT from 1999 to 2012.
- Project Type: Projects are classified into road, bridge, building, mixed project, monitoring system implementation, and others

8.2 Proof of Proposition 1

Proof. Let $\{p_i(\alpha),\, q_i(\alpha)\}$ be the solutions to this constrained optimization problem of a bidder i, such that:

$$\begin{aligned}
\{p_i(\alpha),\ q_i(\alpha)\} &= \arg\max_{p_i,q_i} \ \pi_i^{int} \quad \text{subject to } \ p_i = \alpha\, q_i \\
\Leftrightarrow \quad \{\alpha\, q_i(\alpha),\ q_i(\alpha)\} &= \arg\max_{q_i} \ G_i(\alpha, \boldsymbol{\psi_{-i}})\,(\alpha q_i - vc_i\, C(q_i) - fc_i) \\
\Leftrightarrow \quad \{\alpha\, q_i(\alpha),\ q_i(\alpha)\} &= \arg\max_{q_i} \ \alpha q_i - vc_i\, C(q_i) - fc_i. \qquad (19)
\end{aligned}$$

The first-order necessary condition w.r.t. q_i gives:

$$\begin{aligned}
\alpha - vc_i\, C_q(q_i) &= 0 \\
\Rightarrow \quad q_i(\alpha) &= C_q^{-1}(\alpha/vc_i) \\
\Rightarrow \quad p_i(\alpha) &= \alpha\, C_q^{-1}(\alpha/vc_i).
\end{aligned}$$

It is clear that the second-order condition for maximum is satisfied. Therefore, the pricing and design strategies above indeed is the optimal strategies given $p_i/q_i = \alpha$. □

8.3 Proof of Proposition 2: Multiplicative separability of partially reduced payoff function

I show that $p_i(b_i) - vc_i\, C(q_i(b_i)) - fc_i$ is mutiplicatively separable in vc_i and $e_i \equiv fc_i / C_q^{-1}(1/vc_i)$ given Assumption 1.

Proof. From Proposition 1, it follows that:

$$p_i(b_i) - vc_i\, C(q_i(b_i)) - fc_i \;=\; b_i C_q^{-1}(b_i/vc_i) - vc_i\, C(C_q^{-1}(b_i/vc_i)) - fc_i.$$

Now, Assumption 1 implies that $C_q^{-1}(.)$ is homogeneous of degree $1/(\gamma - 1)$. Therefore, it follows that:

$$\begin{aligned} b_i C_q^{-1}(b_i/vc_i) - vc_i\, C(C_q^{-1}(b_i/vc_i)) - fc_i &= vc_i^{-\frac{1}{\gamma-1}}(b_i C_q^{-1}(b_i) - C(C_q^{-1}(b_i)) - e) \\ &= C_q^{-1}(1/vc_i)\,(u(b_i) - e_i), \end{aligned}$$

where $u(b_i) \equiv b_i\, C_q^{-1}(b_i) - C(C_q^{-1}(b_i))$, and $e_i = fc_i\, vc_i^{\frac{1}{\gamma-1}} \equiv fc_i / C_q^{-1}(1/vc_i)$. Therefore, the ex-post winning payoff is multiplicatively separable into a function of vc_i and a function of e_i. $\square$

8.4 Proof of Proposition 3: Equilibrium existence, monotonicity, and continuity

I first show monotonicity of equilibrium PQR strategy in e_i assuming that an equilibrium exists. Suppose, contrary to the claim, that there exists a non-monotone equilibrium, such that there exist some $\psi_1(e_1) \equiv \psi_1 > \psi_2 \equiv \psi_2(e_2)$ with $e_1 < e_2$. Since bidder i has no better alternative than ψ_i in equilibrium, we have:

$$\begin{aligned} G_i(\psi_1, \boldsymbol{\psi_{-i}})(u(\psi_1) - e_1) &\geq G_i(\psi_2, \boldsymbol{\psi_{-i}})(u(\psi_2) - e_1) \\ G_i(\psi_2, \boldsymbol{\psi_{-i}})(u(\psi_2) - e_2) &\geq G_i(\psi_1, \boldsymbol{\psi_{-i}})(u(\psi_1) - e_2), \end{aligned}$$

which implies:

$$
\begin{aligned}
& G_i(\psi_1, \boldsymbol{\psi_{-i}})(u(\psi_1) - e_1) - G_i(\psi_2, \boldsymbol{\psi_{-i}})(u(\psi_2) - e_1) \\
\geq \;& G_i(\psi_1, \boldsymbol{\psi_{-i}})(u(\psi_1) - e_2) - G_i(\psi_2, \boldsymbol{\psi_{-i}})(u(\psi_2) - e_2) \\
\Rightarrow \quad & (G_i(\psi_2, \boldsymbol{\psi_{-i}}) - G_i(\psi_1, \boldsymbol{\psi_{-i}}))e_1 \geq (G_i(\psi_2, \boldsymbol{\psi_{-i}}) - G_i(\psi_1, \boldsymbol{\psi_{-i}}))e_2 \\
\Rightarrow \quad & e_1 \geq e_2 \quad \because G_i(\psi_2, \boldsymbol{\psi_{-i}}) > G_i(\psi_1, \boldsymbol{\psi_{-i}}),
\end{aligned}
$$

which is a contradiction. Therefore, PQR strategy is non-decreasing in e_i for any equilibrium if exists.

Now, I prove existence. Multiplicative separability of the partially reduced form payoff fuction implies that the equilibrium PQR strategy of a bidder is independent of vc_i because:

$$
\begin{aligned}
\max_{b_i \in [0,B]} \quad \pi_i^{int} &= \max_{b_i \in [0,B]} \quad G_i(b_i, \boldsymbol{\psi_{-i}})\,(u(b_i) - e_i) \\
&= \max_{b_i \in [0,B]} \quad ln(G_i(b_i, \boldsymbol{\psi_{-i}})) + ln(u(b_i) - e_i).
\end{aligned}
$$

To show the existence of monotone equilibrium, I show log-supermodularity between own bids b_i and private information e_i. Then, I apply existence theorem proposed in Athey (2001), such that:

$$
\begin{aligned}
\psi_i(e_i) &\equiv \arg\max_{b_i \in [0,B]} \quad G_i(b_i, \boldsymbol{\psi_{-i}})\,(p_i(b_i) - vc_i\, C(q_i(b_i)) - fc_i) \\
&= \arg\max_{b_i \in [0,B]} \quad C_q^{-1}(1/vc_i)\, G_i(b_i, \boldsymbol{\psi_{-i}})\,(u(b_i) - e_i) \\
&= \arg\max_{b_i \in [0,B]} \quad G_i(b_i, \boldsymbol{\psi_{-i}})\,(u(b_i) - e_i) \qquad \forall \;\; i \in \mathcal{N}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 ln(\pi_i^{int})}{\partial b_i \partial e_i} &= \frac{\partial^2 ln(u(b_i) - e_i)}{\partial b_i \partial e_i} \\
&= \frac{u_b(b_i)}{(u(b_i) - e_i)^2} \\
&> 0,
\end{aligned}
$$

which completes the proof of equilibrium existence.

If ψ_i is not continuous, then there exists e^* with

$$
\lim\ \sup{}_{e<e^*} \psi_i(e) \quad < \quad \lim\ \inf{}_{e>e^*} \psi_i(e).
$$

By Assumption 2, $G_i(.,.)$ is continuous in the first argument. Thus, discontinuity in $\psi_i(.)$ implies discontinuity in the probability of winning function $G_i(.,.)$, such that:

$$\lim\ \sup{}_{e<e^*} G_i(\psi_i(e), \boldsymbol{\psi_{-i}}) \quad > \quad \lim\ \inf{}_{e>e^*} G_i(\psi_i(e), \boldsymbol{\psi_{-i}}).$$

Now, bidder i's expected payoff must be continuous around e^*in equilibrium, and therefore $\lim\ \sup\ G_i(\psi_i(e) - \delta, \boldsymbol{\psi_{-i}})$ must be very close to $\lim\ \inf\ G_i(\psi_i(e) + \delta, \boldsymbol{\psi_{-i}})$ for arbitrarily small $\delta > 0$. However, this is impossible since $\lim\ \sup\ G_i(\psi_i(e), \boldsymbol{\psi_{-i}})$ is strictly greater than $\lim\ \inf\ G_i(\psi_i(e), \boldsymbol{\psi_{-i}})$, and so no bidder would bid $\lim\ \inf\ \psi_i(e)+\delta$. Thus, $\psi_i(.)$ is continuous.

8.5 Proof of Proposition 4:

Proof. Consider equation (4). By implicitly differentiating with respect to τ, I have:

$$\frac{d\,\psi_i(e_i)}{d\,\tau} \quad = \quad \frac{d\,q_i^{\psi}(c_i)}{d\,\tau} C_{qq}(q_i^{\psi}(c_i))\,vc_i, \tag{20}$$

which implies that $\frac{d\,q_i^{\psi}(c_i)}{d\,\tau}$ and $\frac{d\,\psi_i(e_i)}{d\,\tau}$ have the same sign. Also,

$$\frac{d\,p_i^{\psi}(c_i)}{d\,\tau} \quad = \quad \frac{d\,\psi_i(e_i)}{d\,\tau} q_i^{\psi}(c_i) + \frac{d\,q_i^{\psi}(c_i)}{d\,\tau} \psi_i(c_i). \tag{21}$$

Therefore,

$$sign\left(\frac{d\,\psi_i(e_i)}{d\,\tau}\right) = sign\left(\frac{d\,p_i^{\psi}(c_i)}{d\,\tau}\right) = sign\left(\frac{d\,q_i^{\psi}(c_i)}{d\,\tau}\right), \tag{22}$$

which completes the proof. □

The rest of this section is organized as follows. First, I consider a model with neither auction nor reviewer heterogeneities. I show that normalizing the quality cost function C(.) is required even in this simplest case. Next, both observed and unobserved auction heterogeneities are introduced to the simple model, and I derive its implications on the equilibrium strategies of bidders. Finally, unobserved reviewer heterogeneity and evaluation noise is introduced. It is shown that the distributions of unobserved auction, bidder, and reviewer heterogeneities together with evaluation uncertainty are all semiparametrically identified under a normalized quality cost function.

8.6 Non-identification of quality cost function

To demonstrate the identification problem, I start with the simplest case in which there is neither auction nor reviewer heterogeneity. That is, I consider the identification problem under identical auctions with perfectly observed design quality. Suppose that the econometrician observes a random sample of A auctions, indexed by a , with the following information for each auction: price and design quality, $\{p_{ia}, q_{ia} : i = 1, 2, .., N_a\}$, where i is the bidder subindex, and N_a is the number of bidders in auction a.

Given that design quality is observable, the econometrician observes the joint distribution of price and design quality, $F_{p,q}$. Thus, the question is whether or not the econometrician is able to recover the functions of the primitives $C(.), F_{e,vc}$ from the observed distribution of bids $F_{p,q}$. It turns out that $C(.), F_{e,vc}$ cannot be separately identified from the observed bids. To see this, consider the boundary condition where the most inefficient type, $\bar{e} \equiv \bar{f}c/C_q^{-1}(1/\bar{v}c)$, makes zero ex-post payoff with no chance of winning.If the most inefficient type makes a positive ex-post payoff in a continuous monotone equiilibrium, then there is always a small deviation in a PQR bid that gives the bidder a positive chance of winning with a positive ex-post payoff, violating the equilibrium condition. Let $\{\bar{b},\, \bar{p},\, \bar{q}\}$ denote the PQR, price, and design quality submitted by type \bar{e} . Since $G_i(\bar{b}, \boldsymbol{\psi_{-i}}) = 0$, the first order conditions for the boundary type are given by:

$$\begin{aligned} \bar{b} &= \bar{v}c\, C_q(\bar{q}) \\ \bar{p} - \bar{v}c\, C(\bar{q}) - \bar{f}c &= 0. \end{aligned}$$

It is clear from the above conditions that two equations cannot pin down four unknowns, $\{\bar{v}c,\, \bar{f}c,\, C_q(\bar{q}),\, C(\bar{q})\}$. Even if a parameteric assumption on $C(.)$ is imposed where $C(.)$ can be identified from $C_q(.)$, the identification fails.

8.7 Proof of Proposition 5: Multiplicative separability of bidding strategies

Let $b(\theta_a, e_{ia})$ be the equilibrium PQR strategy of bidder i in an auction a with unobserved heterogeneity θ_a with efficiency private information e_{ia}. Similarly, define $s_{ia} \equiv b(0, e_{ia})$, such that s_{ia} is the strategy of bidder i when $\theta_a = 0$. I show that there is a function of θ_a, say $h(\theta_a)$, that satisfy $b(\theta_a, e_{ia}) = h_s(\theta_a)\, s_{ia}$. Then, I show that the two first order optimality conditions are satisfied under the conjectured equilibrium strategy profile. For the sake of simplicity, I omit observed heterogeneity in the following proof, but the proof for observed heterogeneity follows exactly the same steps as the proof for unobserved heterogeneity shown below.

Proof. The probability of winning function $G_i(.)$ is homogeneous of degree 0, and so I have $G_i(h(\theta_a)\, s_{ia}, h(\theta_a)\boldsymbol{\psi_{-i}}) = G_i(s_{ia}, \boldsymbol{\psi_{-i}})$. Further, its density function $g(.)$ is homogeneous of degree -1 since $G_i(.)$ is homogeneous of degree 0, implying that $g(h(\theta_a)\, s_{ia}) = s_{ia}/h(\theta_a)$. Therefore, the left hand side of (14) can be factored as follows.

$$\begin{aligned} & \kappa(\gamma)\left((h(\theta_a)\, s_{ia})^{\tilde{\gamma}} + \tilde{\gamma}\,(h(\theta_a)\, s_{ia})^{\tilde{\gamma}-1}\frac{G_i(h(\theta_a)\, s_{ia}, h(\theta_a)\boldsymbol{\psi_{-i}})}{g_i(h(\theta_a)\, s_{ia}, h(\theta_a)\boldsymbol{\psi_{-i}})}\right) \\ = \; & \kappa(\gamma)\, h(\theta_a)^{\tilde{\gamma}}\left({s_{ia}}^{\tilde{\gamma}} + \tilde{\gamma}\, {s_{ia}}^{\tilde{\gamma}-1}\frac{G_i(s_{ia}, \boldsymbol{\psi_{-i}})}{g_i(s_{ia}, \boldsymbol{\psi_{-i}})}\right). \end{aligned}$$

Therefore, $h(\theta_a) = \exp\{\frac{\gamma-1}{\gamma}\,\rho\,\theta_a\}$ does not affect the first order condition (14) in any way.

Now, consider pricing strategy. It is immediate that pricing strategy is log-linear in θ_a, such that:

$$\begin{aligned} \dot{p}(Z_a, \theta_a, \varepsilon^e_{ia}, \varepsilon^v_{ia}) &= \tilde{\gamma}\,(\frac{1}{\tilde{\gamma}}\,\rho\,\theta_a + \dot{s}_{ia}) + (1-\rho)\,\theta_a + \varepsilon^v_{ia} - (\tilde{\gamma}-1)\,\dot{\gamma} \\ &= \theta_a + \tilde{\gamma}\,\dot{s}_{ia} + \varepsilon^v_{ia} - (\tilde{\gamma}-1)\,\dot{\gamma}, \end{aligned}$$

which completes the proof. □

8.8 Proof of Proposition 7: Equilibrium uniqueness in PQR auction

Proof. By Proposition 3, equilibrium exists and equilibrium PQR strategy, $\psi_i(e_i)$, is non-decreasing and continuous. Here, I show that the equilibrium bidding strategy $\psi_i(e_i)$ is strictly increasing and differentiable. Lastly, I prove the uniqueness of the equilibrium bidding strategy.

Suppose that $\psi_i(e) = \psi_i'$ on the interval $[e^{(1)}, e^{(2)}]$. Given that equilibrium bidding strategy is non-decreasing, it is obvious that $\psi_i(e) < \psi_i'$ for $e < e^{(1)}$, and $\psi_i(e) > \psi_i'$ for $e > e^{(2)}$. Then, a bidder of type $e^{(1)}$ faces the probability of winning $\mathcal{P}$, which can be expressed as:

$$\mathcal{P} \;=\; \sum_{k=0}^{N-1}\frac{1}{k}\binom{N-1}{k}(F_e(e^{(2)}) - F_e(e^{(1)}))^k(1-F(e^{(1)}))^{N-1-k}.$$

where N is the number of bidders. Bidder i of type $e^{(1)}$ can, however, bid $\psi_i' - \delta$ for arbitrarily small $\delta > 0$, and its probability of winning is at least $(1-F_e(e^{(1)}))^{N-1}$. Thus, a bidder i of type $e^{(1)}$ faces a discrete increase in the probability of winning, which is at least $(1-F_e(e^{(1)}))^{N-1} - \mathcal{P}$, and so ψ_i' cannot be an equilibrium bid for $e^{(1)}$.

Non existence of asymmetric equilibrium directly follows from Maskin and Riley (1982), and therefore I impose symmetry on the bidding strategy (i.e., $\psi_i(.) = \psi(.)$). Given symmetric strictly increasing bidding strategy, the probability of winning function, $G_i(\psi_i(e_i), \boldsymbol{\psi_{-i}})$, can be written as:

$$G_i(\psi_i(e_i), \boldsymbol{\psi_{-i}}) \quad = \quad (1 - F_e(e_i))^{N-1}$$

To avoid cluttering, I denote the probability of winning $(1 - F_e(e_i))^{N-1}$ by $G(e_i)$ from this point on. To see that the equilibrium bidding strategy $\psi(e_i)$ is differentiable, consider two distinct types $e^{(1)}$ and $e^{(2)}$ with $e^{(2)} = e^{(1)} + \delta$ where $\delta > 0$. Equilibrium conditions imply:

$$\begin{aligned} G(e^{(1)})\,(u(\psi(e^{(1)})) - e^{(1)}) \quad &\geq \quad G(e^{(2)})\,(u(\psi(e^{(2)})) - e^{(1)}) \\ G(e^{(2)})\,(u(\psi(e^{(2)})) - e^{(2)}) \quad &\geq \quad G(e^{(1)})\,(u(\psi(e^{(1)})) - e^{(2)}) \end{aligned}$$

The above two inequality can be rewritten as:

$$\begin{aligned} (G(e^{(1)}) - G(e^{(2)}))\,(u(\psi(e^{(1)})) - e^{(1)}) \quad &\geq \quad G(e^{(2)})\,(u(\psi(e^{(2)})) - u(\psi(e^{(1)}))) \\ (G(e^{(2)}) - G(e^{(1)}))\,(u(\psi(e^{(2)})) - e^{(2)}) \quad &\geq \quad G(e^{(1)})\,(u(\psi(e^{(1)})) - u(\psi(e^{(2)}))) \end{aligned}$$

By the mean value theorem, there exist $\psi^* \in (\psi(e^{(1)}), \psi(e^{(2)}))$ and $\psi^{**} \in (\psi(e^{(1)}), \psi(e^{(2)}))$, such that:

$$\begin{aligned} (G(e^{(1)}) - G(e^{(2)}))\,(u(\psi(e^{(1)})) - e^{(1)}) \quad &\geq \quad G(e^{(2)})\,(\psi(e^{(2)}) - \psi(e^{(1)}))\,\psi^* \\ (G(e^{(2)}) - G(e^{(1)}))\,(u(\psi(e^{(2)})) - e^{(2)}) \quad &\geq \quad G(e^{(1)})\,(\psi(e^{(1)}) - \psi(e^{(2)}))\,\psi^{**} \end{aligned}$$

Rearranging the above two inquality, I have:

$$\frac{(G(e^{(1)}) - G(e^{(2)}))\,(u(\psi(e^{(1)})) - e^{(1)})}{G(e^{(2)})\,\psi^*} \geq \quad \psi(e^{(2)}) - \psi(e^{(1)}) \quad \geq \frac{(G(e^{(1)}) - G(e^{(2)}))\,(u(\psi(e^{(2)})) - e^{(2)})}{G(e^{(1)})\,\psi^{**}}$$

Dividing through by δ, and let $\delta \to 0$ gives:

$$\frac{-G'(e^{(1)})\,(u(\psi(e^{(1)})) - e^{(1)})}{G(e^{(1)})\,\psi(e^{(1)})} \geq \quad \lim_{\delta \to 0} \tfrac{\psi(e^{(1)}+\delta) - \psi(e^{(1)})}{\delta} \quad \geq \frac{-G'(e^{(1)})\,(u(\psi(e^{(1)})) - e^{(1)})}{G(e^{(1)})\,\psi(e^{(1)})}$$

where $G'(e^{(1)}) \equiv \partial G(e_i)/\partial e_i$ at $e_i = e^{(1)}$. Since the above inequality holds for any e_i, $\psi(e_i)$ is differentiable and given by:

$$\frac{d\,\psi(e_i)}{d\,e_i} = \frac{-G'(e_i)\,(u(\psi(e_i)) - e_i)}{G(e_i)\,\psi(e_i)} \tag{23}$$

Now, I turn to prove equilibrium uniqueness. It suffices to show that the initial value for the differential equation (23) is unique since the bidding strategy for the other types are uniquely determined by (23). Conjecture that the most inefficient type $\bar{e}$ with $F_e(\bar{e}) = 1$ bids $\psi(\bar{e}) = u^{-1}(\bar{e})$. As a bidder of type $\bar{e}$ makes zero profit when bidding $u^{-1}(\bar{e})$, bidding below $u^{-1}(\bar{e})$ would yields negative payoff, and so any bid lower that $u^{-1}(\bar{e})$ is certainly not equilibrium bidding strategy for type $\bar{e}$. Now, consider a case in which type $\bar{e}$ bids above $u^{-1}(\bar{e})$. Then, there exists a sufficiently small $\delta > 0$, such that $\psi = u^{-1}(\bar{e}) + \delta$, which makes the most inefficient type earn a positive expected profit since the equilibrium strategy $\psi()$ is continuous. This is a contradiction, however, because the most inefficient type cannot win with a positive probabiltiy when equilibrium bidding strategy $\psi(e_i)$ is strictly increasing in e_i. Therefore, $\psi(\bar{e}) = u^{-1}(\bar{e})$ is the unique initial value for the differential equation of (23), which completes the proof of uniqueness.

To see that the unique equilibrium bidding strategy takes the expression given in (18), consider first the expected payoff of bidder i in the unique equilibrium.

$$\pi_i^{int} = G(e_i)(u(\psi(e_i)) - e_i) \tag{24}$$

By the envelope theorem, I have:

$$\frac{d\,\pi_i^{int}}{d\,e_i} = -G(e_i)$$

Integrating back with respect to e_i gives:

$$\pi_i^{int} = \int_{e_i}^{\bar{e}} G(x)\,dx \tag{25}$$

Solving for $\psi(e_i)$ using equations (24) and (25), I obtain:

$$\begin{aligned}\psi(e_i) &= u^{-1}\left(\frac{\int_{e_i}^{\bar{e}} G(x)\,dx}{G(e_i)}\right) \\ &= u^{-1}\left(\frac{\int_{e_i}^{\bar{e}} (1 - F_e(x))^{N-1}\,dx}{(1 - F_e(e_i))^{N-1}}\right)\end{aligned}$$

which is the desired expression.

□

8.9 Equilibrium computation algorithm

Step 1: Set the level of evaluation uncertainty arbitrarily large. Denote the pseudo evaluation uncertainty by τ^j where τ^0 is the initial level of pseudo evaluation uncertainty. Then, I draw 300 types from $e_{ia} \sim \hat{F}_e(.|N, R)$.

Step 2: Guess a strategy of each bidder and denote them by $\psi_i^{j,k}(e_i)$ for $k = 0$. Given the strategy of bidders, compute the equilibrium strategy by applying the following Quasi-Newton map to every bidder of every type simultaneously (Here I suppress the dependency of strategy on $\mathbf{c_i}$ for the sake of visual clarity).

$$\psi_i^{j,k+1} \quad = \quad \psi_i^{j,k} - H^{j,k}\, J^{j,k}, \tag{26}$$

where $J^{j,k}$ and $B^{j,k}$ are the Jacobian and inverse Hessian of π_i^{int}. Since inversion of Hessian matrix is computationally very expensive, I approximate $H^{j,k}$ (say $\hat{H}^{j,k}$) by BFSG method, such that:

$$\hat{H}^{j,k+1} = \left(I - \frac{(J^{j,k+1} - J^{j,k})(\psi_i^{j,k+1} - \psi_i^{j,k})^T}{(J^{j,k+1} - J^{j,k})^T(\psi_i^{j,k+1} - \psi_i^{j,k})} \right)^T \hat{H}^{j,k} \left(I - \frac{(J^{j,k+1} - J^{j,k})(\psi_i^{j,k+1} - \psi_i^{j,k})^T}{(J^{j,k+1} - J^{j,k})^T(\psi_i^{j,k+1} - \psi_i^{j,k})} \right) + \frac{(\psi_i^{j,k+1} - \psi_i^{j,k})(\psi_i^{j,k+1} - \psi_i^{j,k})^T}{(J^{j,k+1} - J^{j,k})^T(\psi_i^{j,k+1} - \psi_i^{j,k})}.$$

A necessary condition for a Bayesian Nash Equilibrium is $\psi_i^{j,l+1} = \psi_i^{j,l}$ for all $l > K$ where K is some arbitrarily large integer.

Step 3: Upon convergence, I reduce evaluation uncertainty by $\kappa > 0$, such that $\tau^{j+1} = \tau^j - \kappa$. Then, use equilibrium strategy $\psi_i^{j,k}$ as an initial guess for $\psi_i^{j+1,k}$.

Step 4: Repeat step 2 and 3 till $\tau^j = \hat{\tau}$, such that pseudo evaluation uncertainty meets estimated evaluation uncertainty in the data.